NOAA Technical Memorandum NMFS-PIFSC-15

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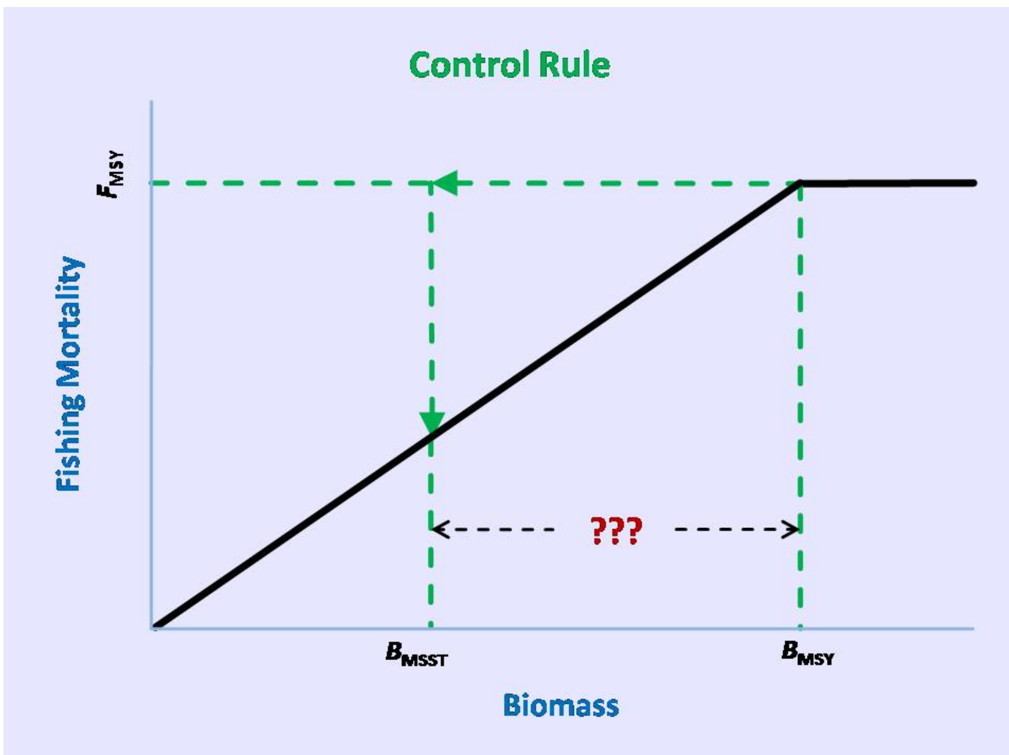
Rationalizing the Formula for Minimum Stock Size Threshold (B_{MSST}) in Management Control Rules



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INTRODUCTION

United States fisheries are managed under National Standard Guidelines (NSGs) that implement National Standard 1 of the Magnuson-Stevens Fishery Conservation and Management Act (MSA). Restrepo et al. (1998) give technical guidelines for constructing fishery management plans (FMPs) that conform to the 1996 MSA NSGs. Much of the advice concerns definition of "control rules" which stipulate which management actions to take depending on whether overfishing is occurring and whether the population is overfished. Under the MSA, a fishery is typically managed with the objective of harvesting, on average, the maximum sustainable yield (MSY). In a fishery achieving this goal, the stock biomass will fluctuate around an average biomass level defined as B_{MSY}. The stock is said to be overfished when the stock biomass falls below a critical level, or reference point, called the "minimum stock size threshold" (B_{MSST}), where $B_{\mathrm{MSST}} < B_{\mathrm{MSY}}$. Even in a well managed fishery, the stock biomass will undergo natural fluctuations above and below the management target. Therefore, it would seem reasonable to allow abundance to decline temporarily below B_{MSY} without launching regulatory actions as long as it doesn't fall below the B_{MSST} threshold.

Following guidance on the default definition of B_{MSST} (Restrepo *et al.* 1998), many control rules define B_{MSST} as a proportion $(1-M)$ of B_{MSY} as follows

$$B_{\mathrm{MSST}} = (1-M)B_{\mathrm{MSY}} \tag{1}$$

where M, the natural mortality, is a measure of population turnover. In theory, populations with fast turnover rates are expected to fluctuate more than those with slow turnover rates, and to have the potential for faster recovery from a depleted state. Thus it is appropriate to allow larger excursions below B_{MSY} for populations with higher values of M as is done by Equation (1).

That formula is problematic in several ways. A proportion should be a unitless figure, but M is measured in units of inverse time. Though the units are usually unstated, they are most often presumed to be yr^{-1}. But the units are in fact an arbitrary choice unrelated to the biology of the animal in question or to the characteristics of the fishery, and that choice affects the value assigned to B_{MSST}. Also, because the value "1" in the expression is unitless, the expression makes the mathematical mistake of mixing different units across an arithmetic (addition or subtraction) operator. Finally, a proportion should be a value between 0 and 1, but given that M can be greater than 1, B_{MSST} could be negative, which is nonsense. To overcome that, most control rules arbitrarily stipulate that B_{MSST} can be no lower than $0.5B_{\mathrm{MSY}}$.

A review of a sample of U.S. FMPs (HMSMD 1999, PFMC 2003, WPRFMC 2002) showed that the default formula in Equation (1) is used to define B_{MSST} for almost all species in the sampled FMPs. Also, the control rules often define another reference point B_{FLG} in the same way. This warning point is defined as the proportion $(1-M)$ of the biomass at optimum yield (B_{OPT}). Given its wide use, it is appropriate for us to seek an alternative expression for the proportion in Equation (1) that avoids the above problems, but has the desired behavior with respect to population turnover. In addition the expression should be based on quantitative reasoning rather than just the intuitive notions above. For that purpose we will suppose that when abundance drops below a level, $B_{\mathrm{MSST}} < B_{\mathrm{MSY}}$, we would restrict fishing in some way, and we want B_{MSST} defined such that under the restricted fishing regime the population could be expected to recover from B_{MSST} to B_{MSY} within a set recovery time Δt. We will concentrate on formulae for B_{MSST}. Similar results for B_{FLG} in relation to B_{OPT} can be obtained in the same way.

CASE WITH NO FISHING DURING RECOVERY

The FMPs contain a variety of rules for setting fishing mortality during recovery. For simplicity we will first assume a severe restriction of zero fishing effort until the population recovers to B_{MSY}. Assume logistic population dynamics so that the rate of change of biomass is given by

$$\frac{\partial B}{\partial t} = r\left(1 - \frac{B}{K}\right)B \tag{2}$$

where B is the biomass abundance, r is maximum production rate, K is the "carrying capacity" or equilibrium abundance with no fishing. B_{MSY} for logistic population dynamics is half the carrying capacity. Integrating Equation (2) with $2B_{\mathrm{MSY}}$ substituted for K gives

$$B(t) = \frac{2B_{\mathrm{MSY}}}{1 + e^{r(t_0 - t)}} \;\; ; \quad B(t_0) = B_{\mathrm{MSY}} \tag{3}$$

Equation (3) can describe a population that is depressed to a point $B_{\mathrm{MSST}} < B_{\mathrm{MSY}}$ at a time t prior to time t_0. So from Equation (3) we have for $\Delta t = t_0 - t$

$$B(t) = \frac{2}{1 + e^{r\Delta t}} \, B_{\mathrm{MSY}} \tag{4}$$

Thus the expression for proportion of B_{MSY} in Equation (1) is replaced by a more complex expression, in this case involving r as a measure of population turnover rather than M and explicitly containing the recovery time to B_{MSY}. To examine how Equation (4) relates to Equation (1), we substitute M as an approximation to r[1] in Equation (4) and take the first two terms of its Taylor expansion giving

$$B_{\mathrm{MSST}} \cong \left(1 - \frac{M\Delta t}{2}\right) B_{\mathrm{MSY}} \tag{5}$$

From that we can see that if we were to choose a recovery period, Δt, of 2 years and measure M in inverse years, then the above approximation would be identical with the default formula (1). A plot of B_{MSST} according to formula (4) with $\Delta t = 2$ compares well with the default formula (Fig. 1) up to the break in the default formula at $M = 0.5$. So the default formula seems to carry an implicit recovery period of approximately 2 years for a rule with no fishing during recovery.

CASE WITH FIXED F_{recovery}

Of course many control rules do not stipulate a complete cessation of fishing once B_{MSST} is reached, but allow some level of fishing mortality during the recovery period. In this case the recovery period would be longer than for recovery without fishing. If that level of fishing mortality, F_{recovery}, were to be held constant, the differential Equation (2) would be modified to

$$\frac{\partial B}{\partial t} = r\left(1 - \frac{B}{K}\right)B - F_{\mathrm{recovery}}B \tag{6}$$

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[1] Though r focuses on increases to the population and M on decreases, they are both measures of population turnover, and are therefore expected to be approximately equivalent.

which is the familiar Schaefer production model. Implicit in this model is the fact that F_{recovery} must be set to a level less than F_{MSY}, otherwise the biomass can never recover to B_{MSY}. If we assume that F_{recovery} is set to a fraction γ of F_{MSY} and note that for the Schaefer model F_{MSY} is equal to $r/2$ and K equal to $2B_{\text{MSY}}$, then Equation (6) can be written

$$\frac{\partial B}{\partial t} = r\left(1 - \frac{B}{K}\right)B - \frac{\gamma r}{2}B \;;\quad \gamma < 1 \tag{7}$$

which integrates to

$$B(t) = \frac{(2-\gamma)B_{\text{MSY}}}{1 + (1-\gamma)e^{(1-\gamma/2)r(t_0 - t)}} \quad ; \quad B(t_0) = B_{\text{MSY}} \tag{8}$$

The result corresponding to Equation (4) for a given recovery time Δt is

$$B(t) = \frac{(2-\gamma)}{1 + (1-\gamma)e^{(1-\gamma/2)r\Delta t}}\, B_{\text{MSY}} \tag{9}$$

Note that Equation (9) reverts to Equation (4) for $\gamma = 0$, i.e. zero fishing effort in the recovery period. Approximating Equation (9) with a Taylor expansion, and again substituting M for r, in this case gives

$$B_{\text{MSST}} \cong \left(1 - \frac{(1-\gamma)M\Delta t}{2}\right)B_{\text{MSY}} \tag{10}$$

which reverts to Equation (5) for $\gamma = 0$. If γ is set to 0.75 as is the case in some control rules, then $\Delta t = 8$ would reduce this approximation to Equation (1). However, the default formula is a closer match to the exact formulation (9) with $\Delta t = 6$ (Fig. 2). So if B_{MSST} is set according to the existing default (1), there is an implicit recovery time of approximately 6 years for γ set at 0.75.

CASE WITH F_{recovery} VARYING IN RELATION TO THE RATIO B/B_{MSY}

Some control rules institute recovery regimes whereby fishing effort is reduced by an amount that varies in time depending on abundance. One such scheme is to set F to some fraction α of F_{MSY} multiplied by the ratio of current abundance to some reference abundance, say B_{MSY}. Thus F_{recovery} would vary with abundance as follows

$$F_{\text{recovery}} = \alpha F_{\text{MSY}} B/B_{\text{MSY}} \tag{11}$$

Again noting that F_{MSY} is $r/2$ and B_{MSY} is $K/2$, the dynamics during the recovery period would be given by

$$\frac{\partial B}{\partial t} = r\left(1 - \frac{B}{2B_{\text{MSY}}}\right)B - \frac{\alpha r}{2B_{\text{MSY}}}B^2 \tag{12}$$

with integral equation

$$B(t) = \frac{2B_{\text{MSY}}}{(1+\alpha) + (1-\alpha)e^{r(t_0 - t)}} \quad ; \quad B(t_0) = B_{\text{MSY}}$$

which leads to B_{MSST} defined by

$$B_{\mathrm{MSST}} = \frac{2}{(1+\alpha) + (1-\alpha)e^{r\Delta t}} B_{\mathrm{MSY}} \tag{13}$$

As in the case with a fixed F_{recovery} Equation (13) reverts to (4) for $\alpha = 0$. Comparison with the default formula with $\alpha = 0.75$ (Fig. 3) shows a good approximation of the default formula with $\Delta t = 5$. Thus the default B_{MSST} with this control rule and α set to 0.75 would have an implicit 5-year recovery time.

CONCLUSION

Various formulae could be chosen for calculating B_{MSST} in place of the existing default formula given in Equation (1) above. The choice of formula would depend on the regime chosen for managing fishing mortality during recovery and the time interval chosen as acceptable for recovery. The alternatives examined above (Equations (4), (9), and (13)) do not represent all possibilities. Others could be derived based on other details of how to manage recovery.

The formulae derived here behave as desired in allowing greater excursion of stock biomass below B_{MSY} for populations with higher turnover (high r). B_{MSST} will not be less than zero even for large values of r, and for very low values of r it will approach B_{MSY}, giving little leeway for biomass excursions below B_{MSY} for populations with slow turnover. Furthermore, none of these formulae contain the mathematical *faux pas* of mixing units in an arithmetic expression. Also, the time units can be anything (years, months, minutes), and the formula will give the same result as long as r and Δt are on the same time scale. Finally, the formulae have the appeal of being based on population dynamic theory, thus allowing managers an explicit choice of desired recovery period.

The population dynamic theory used is the relatively simple Schaefer production model. More complex and sophisticated models could be used instead, and probably should be if warrented by the information available for a particular species, in which case a formula for B_{MSST} could be derived based on that information instead of choosing a catch-all default. But recall that these proposed formulae, and the currently used formula, are designed to apply to a default situation where detailed knowledge is lacking. Thus the simpler Schaefer model is appropriate.

In examining the relationship between the existing default formula and the alternatives derived here, we see that the current default formula implies different recovery periods depending on the choice of recovery regime. It would seem preferable to make that tradeoff explicit by use of a formula that does not hide it.

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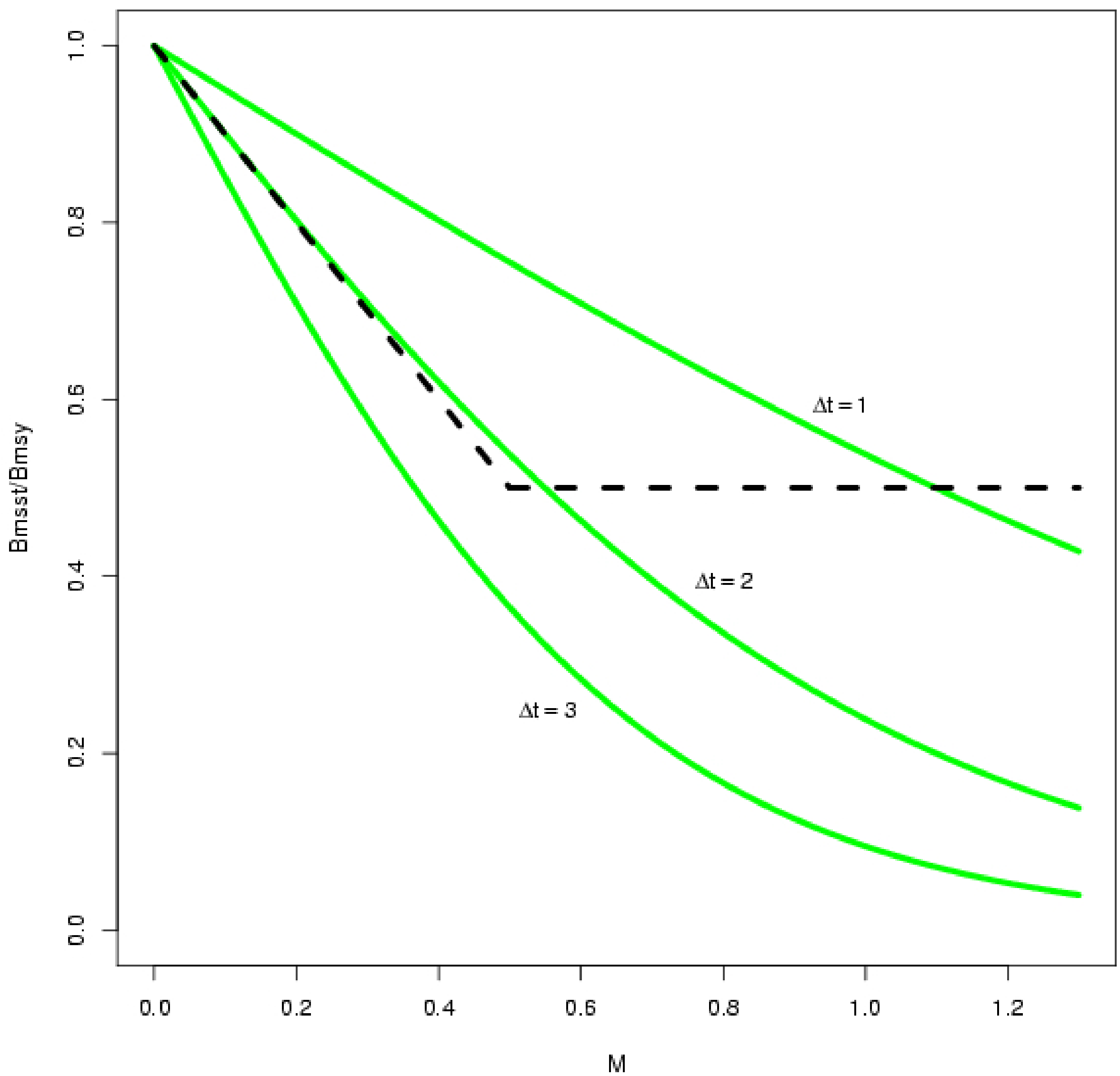

Figure 1. Solid lines: The ratio $B_{\mathrm{MSST}}/B_{\mathrm{MSY}}$ enabling recovery of stock biomass to B_{MSY} after various recovery periods (Δt) for a range of natural mortality (M) when fishing is completely suspended ($F_{\mathrm{recovery}} = 0$). Dashed line: current default.

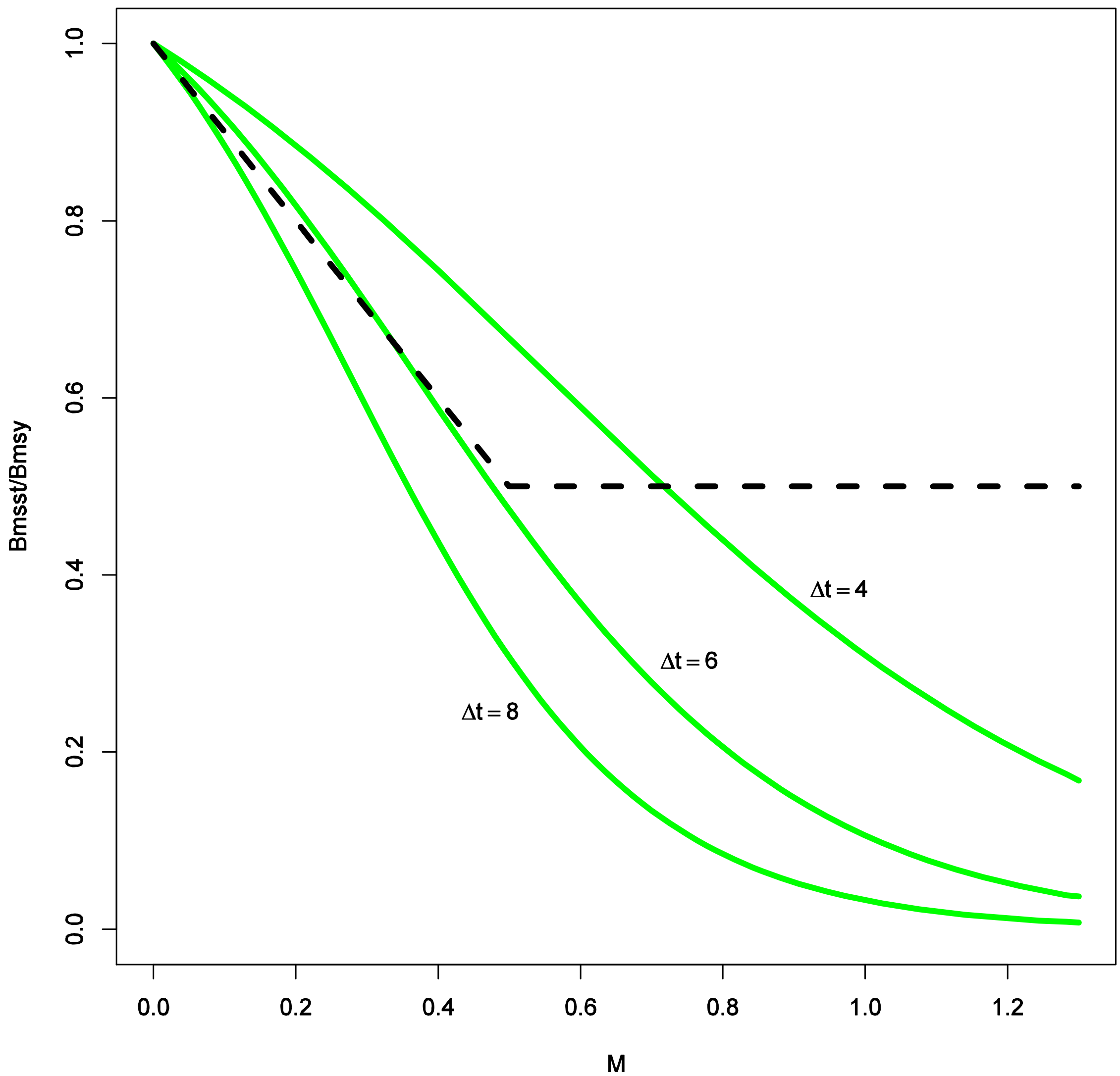

Figure 2. Solid lines: The ratio $B_{\text{MSST}}/B_{\text{MSY}}$ enabling recovery of stock biomass to B_{MSY} after various recovery periods (Δt) for a range of natural mortality (M) when F_{recovery} is fixed at $0.75F_{\text{MSY}}$. Dashed line: current default.

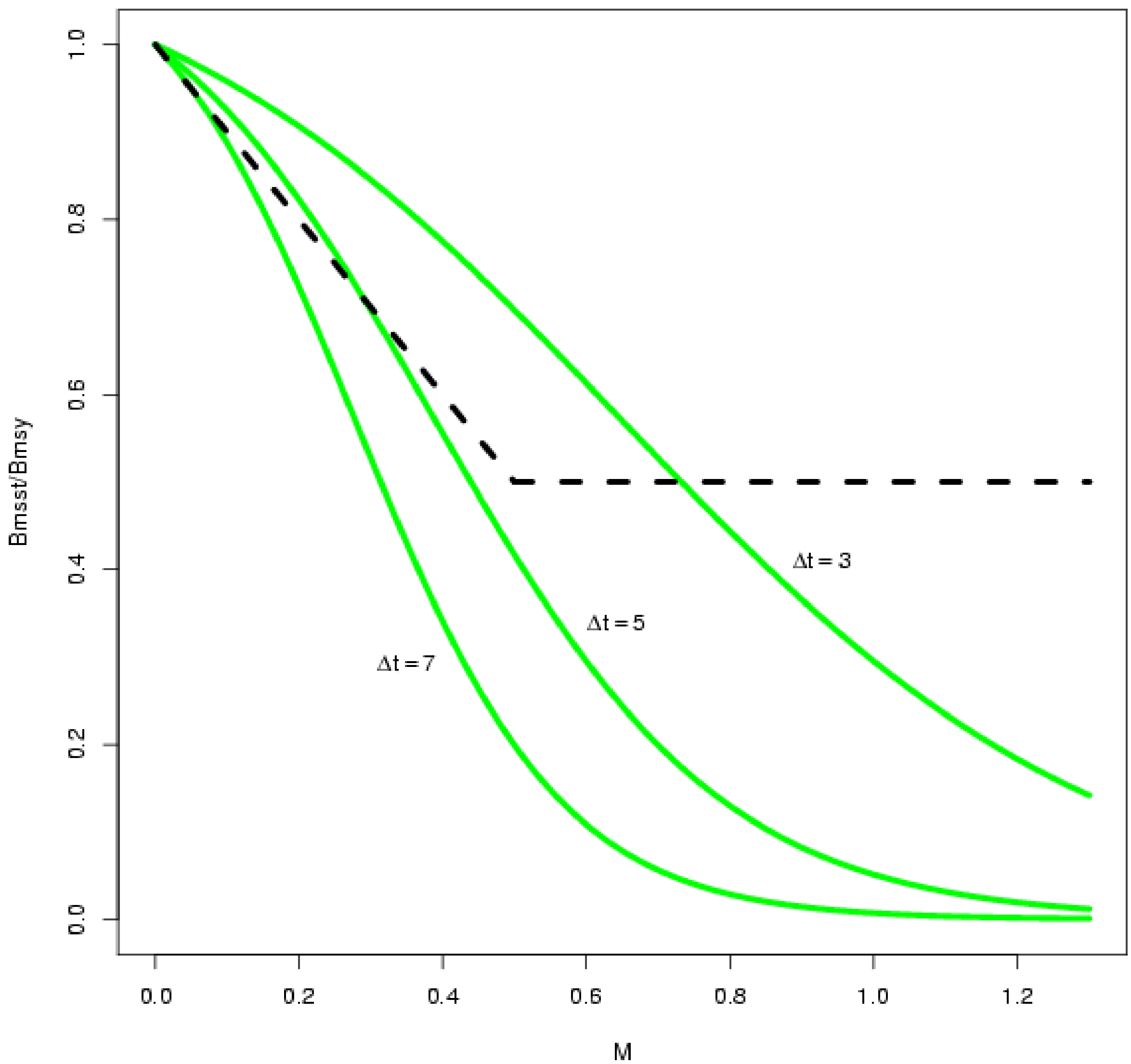

Figure 3. Solid lines: The ratio $B_{\mathrm{MSST}}/B_{\mathrm{MSY}}$ enabling recovery of stock biomass to B_{MSY} after various recovery periods (Δt) for a range of natural mortality (M) when F_{recovery} is varied with stock biomass as $0.75F_{\mathrm{MSY}}\,\mathrm{B}/B_{\mathrm{MSY}}$. Dashed line: current default.

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