With Great Power Comes Great Responsibility: 10 Things to Know About Steepness for Stock Assessments

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Thing #1: How We Write the Beverton-Holt SRR Affects Interpretability

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Comment: What follows can be done for Ricker SRR if that is your preference

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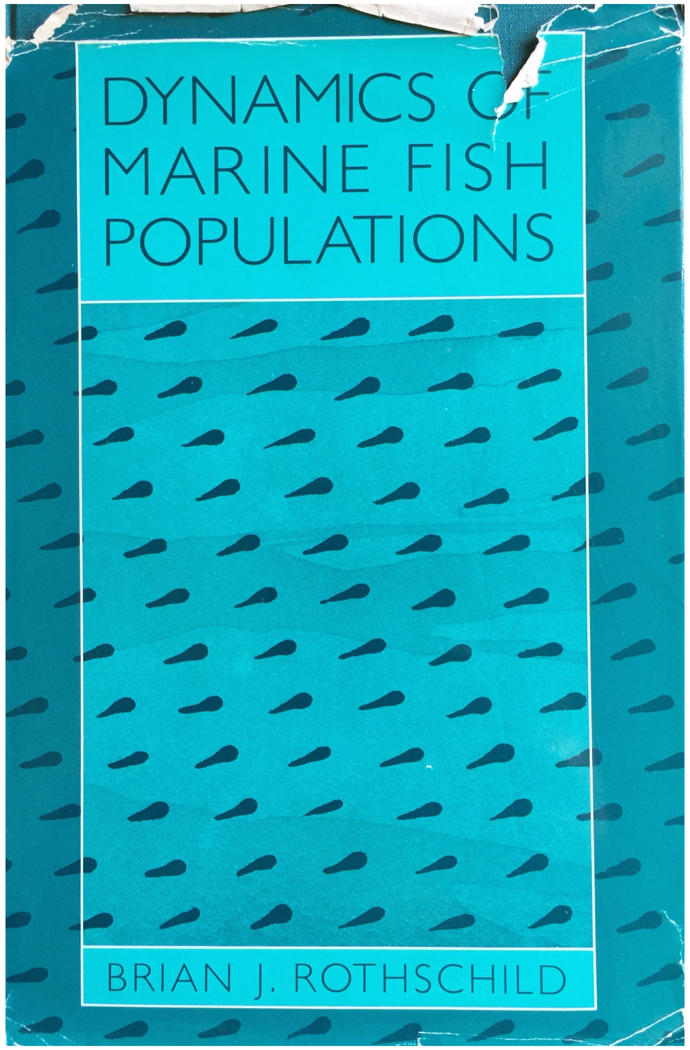

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$\frac{1}{\beta} =$ Egg numbers giving half of maximum egg survival

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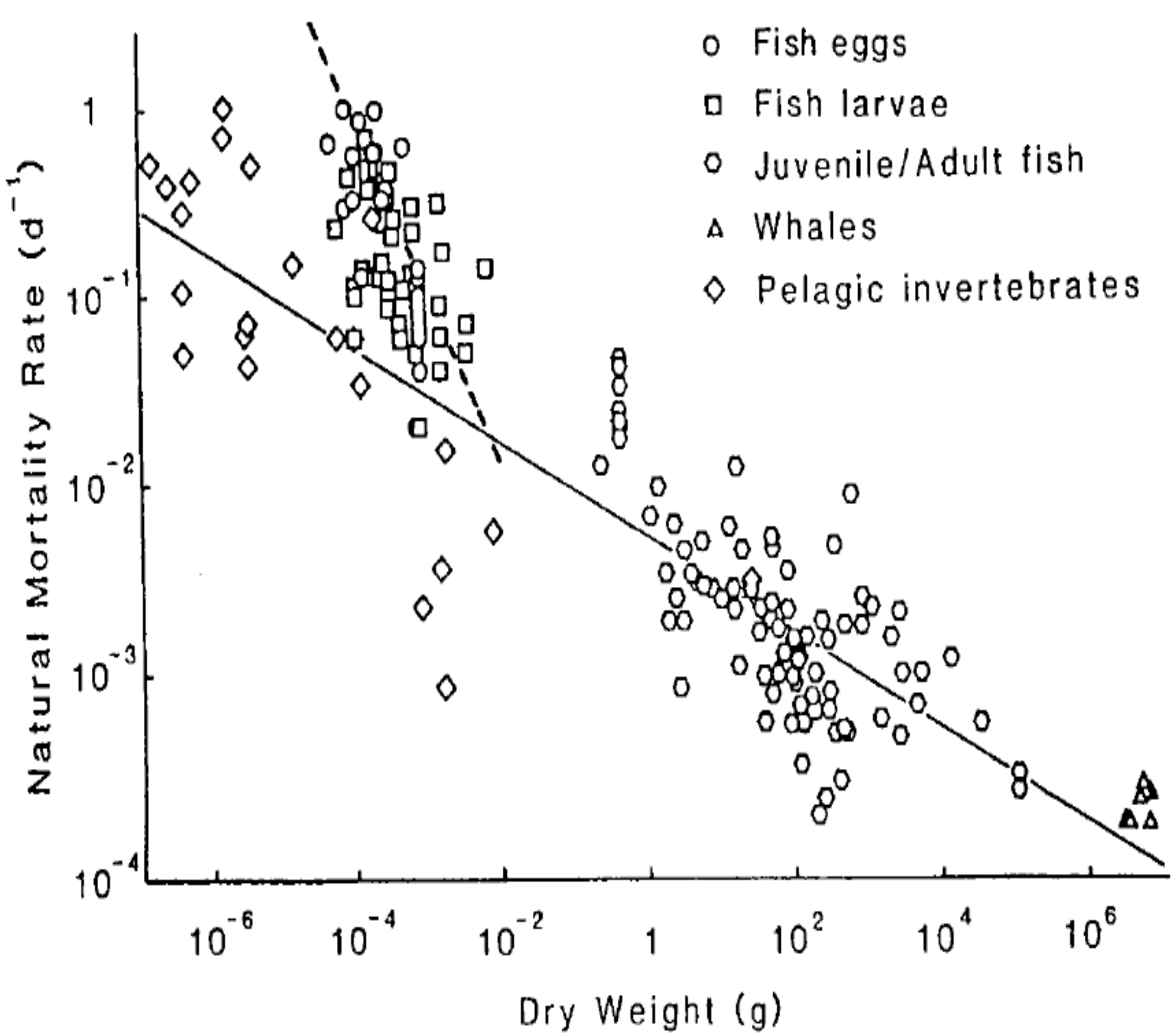

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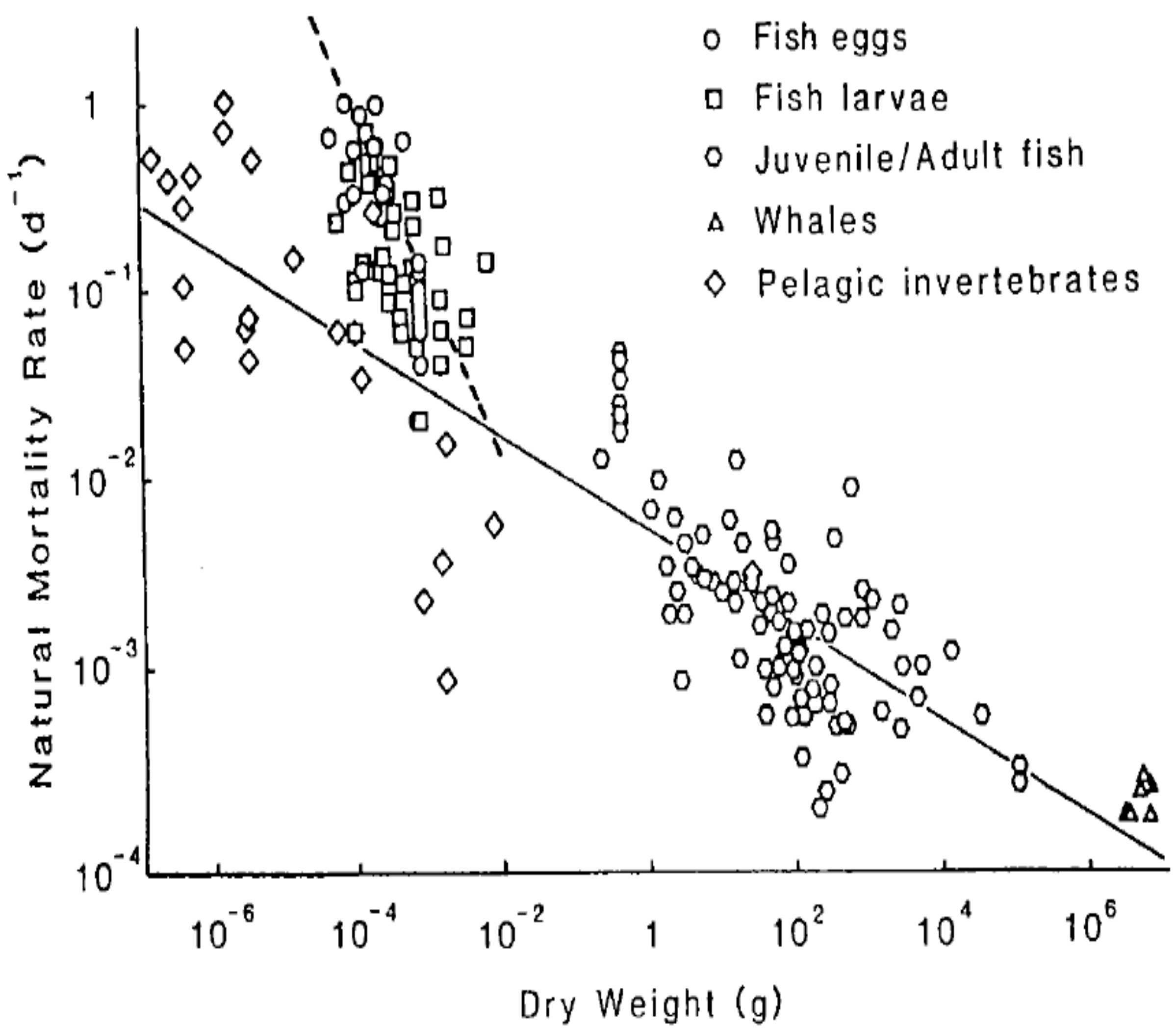

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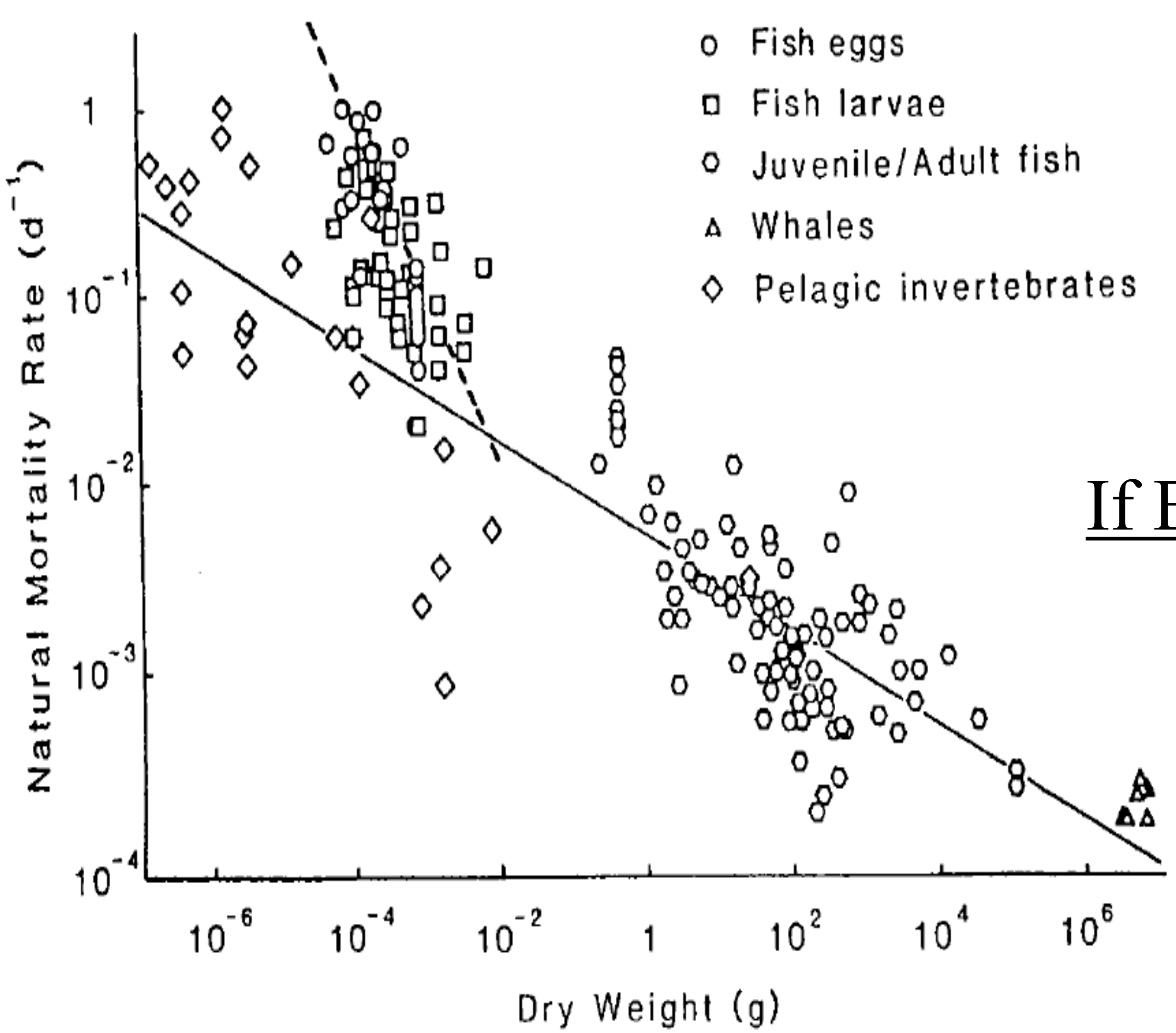

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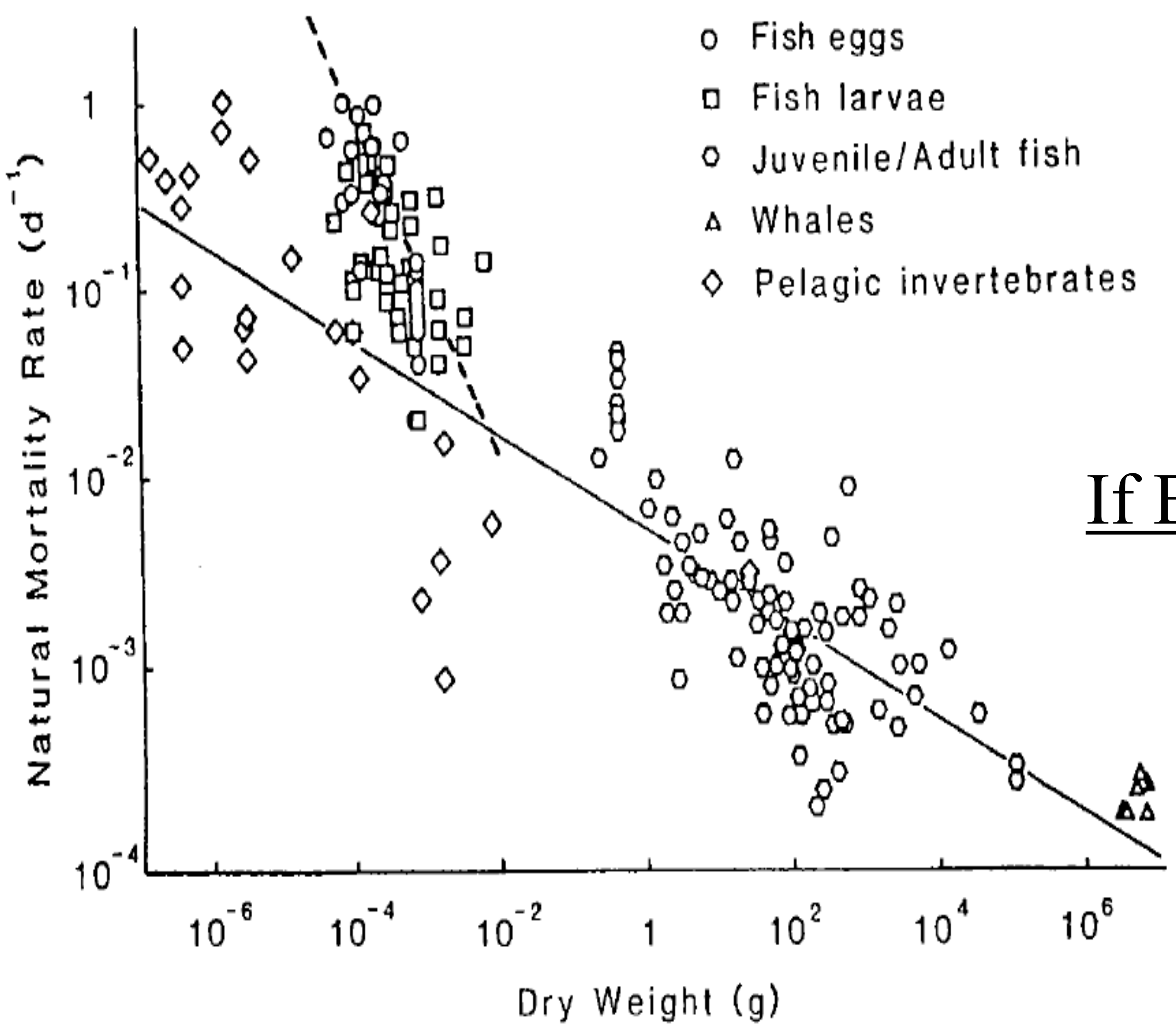

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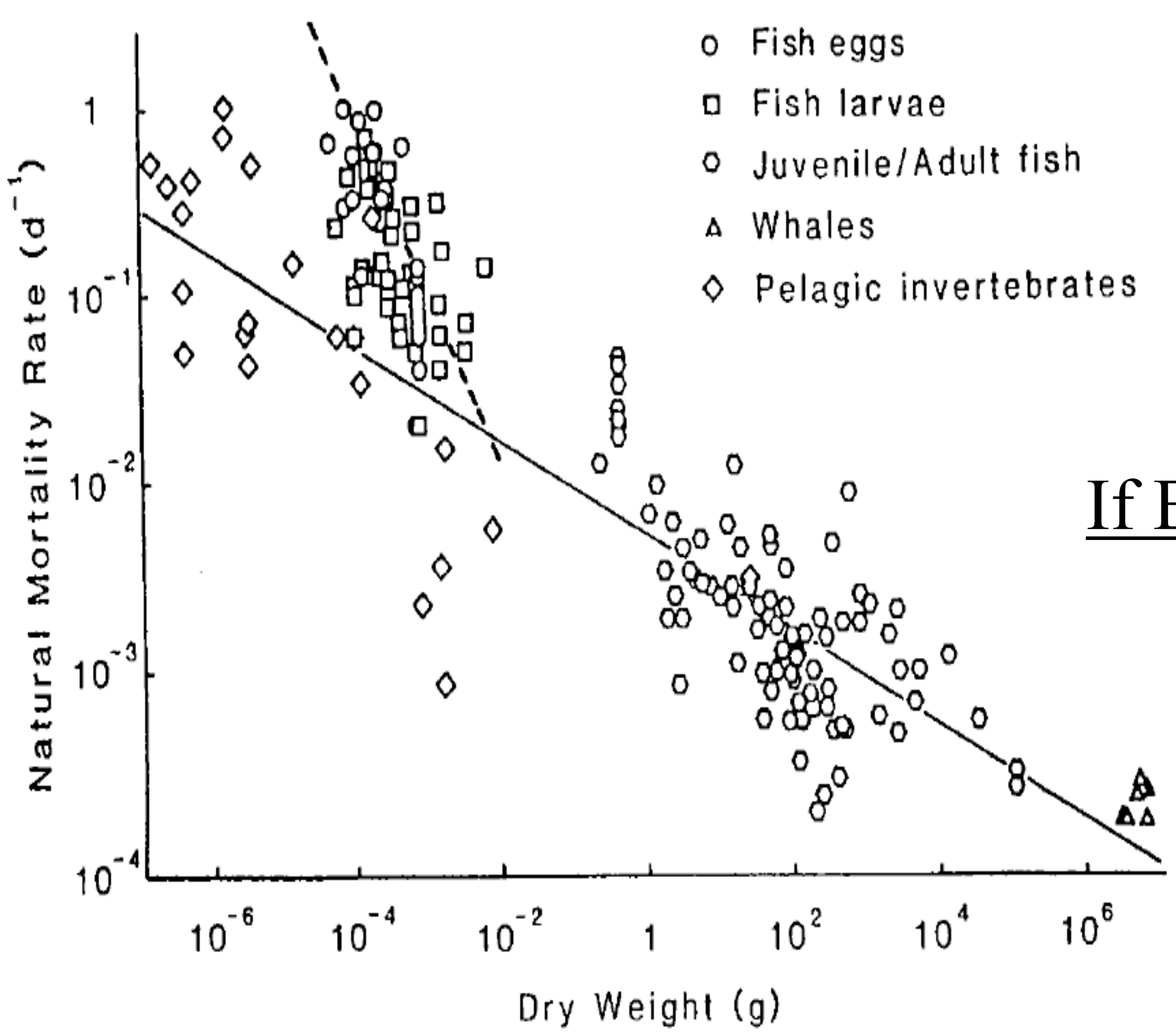

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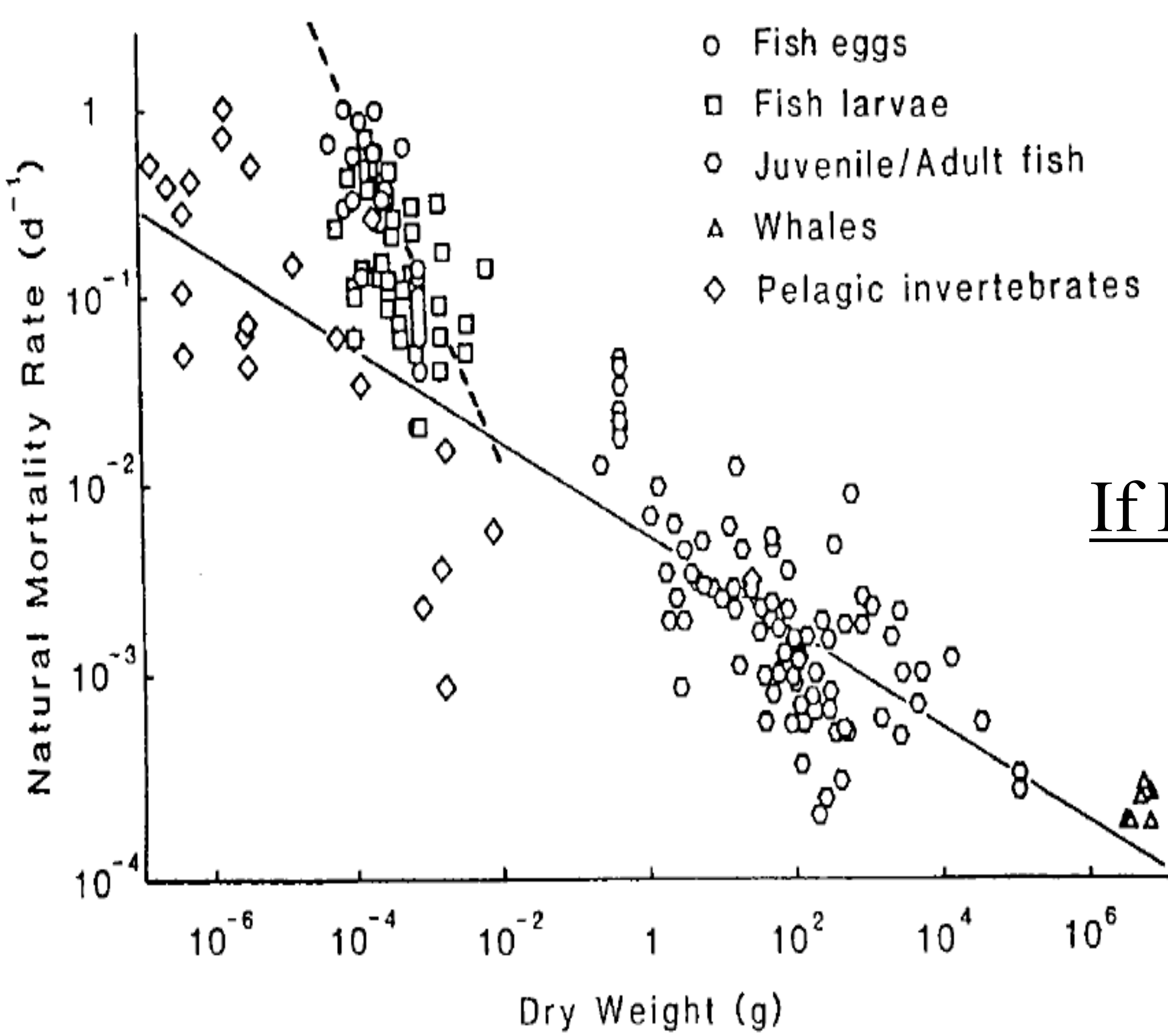

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- Initial egg/larval size
- Size at recruitment

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Life time individual egg production

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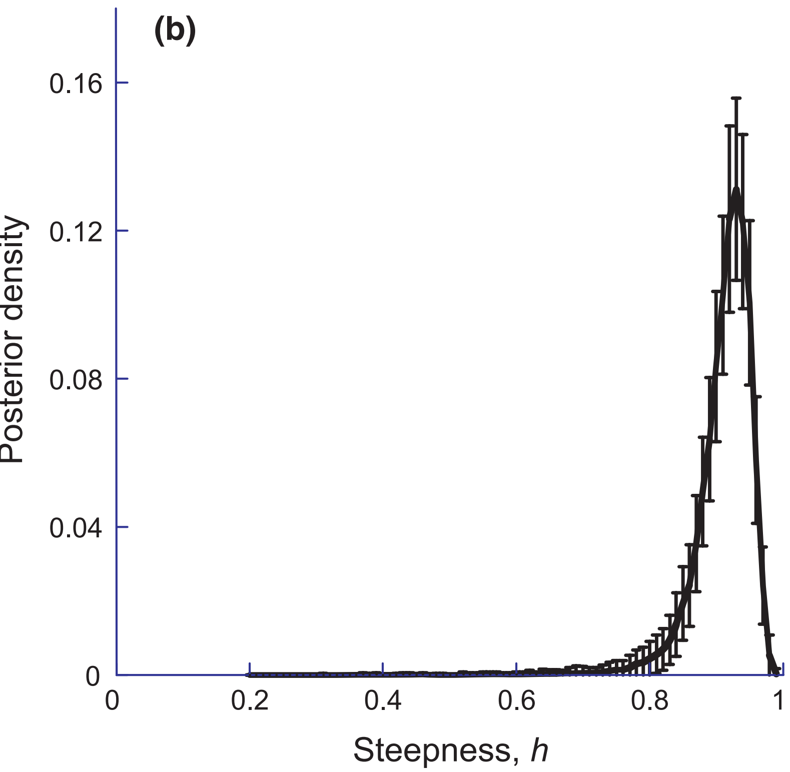

Bluefin tuna
Mangel et al FaF (2010)

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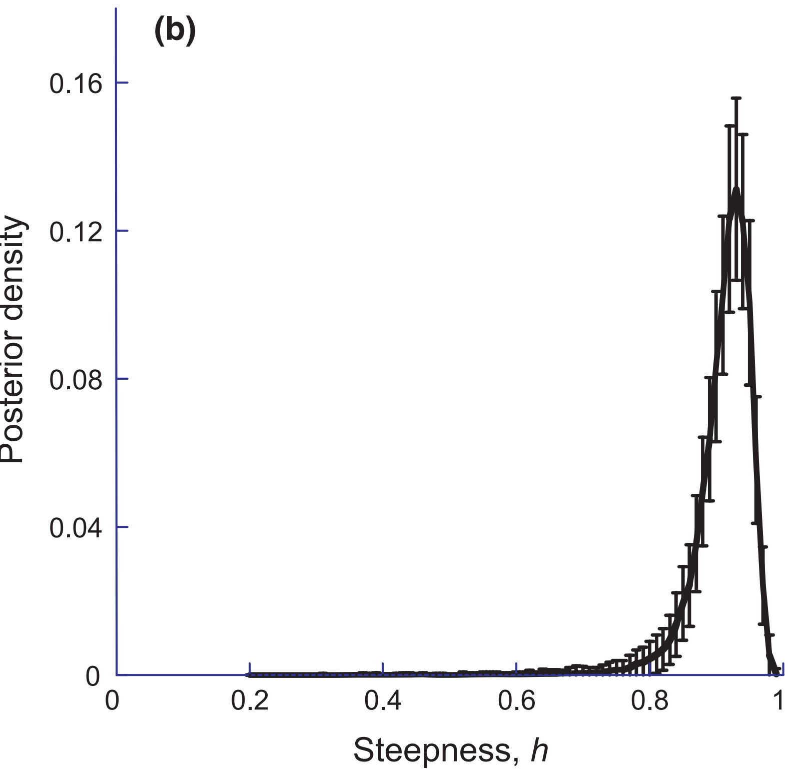

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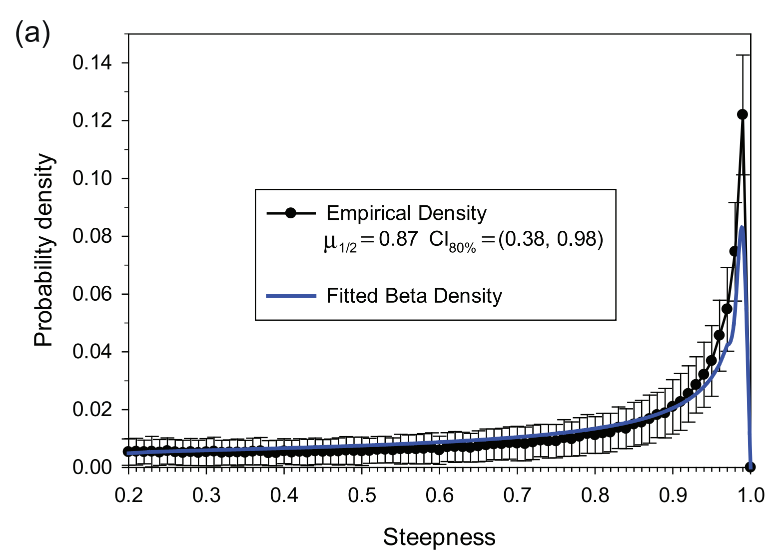

Striped Marlin
Brodziak et al Fish Res 2014

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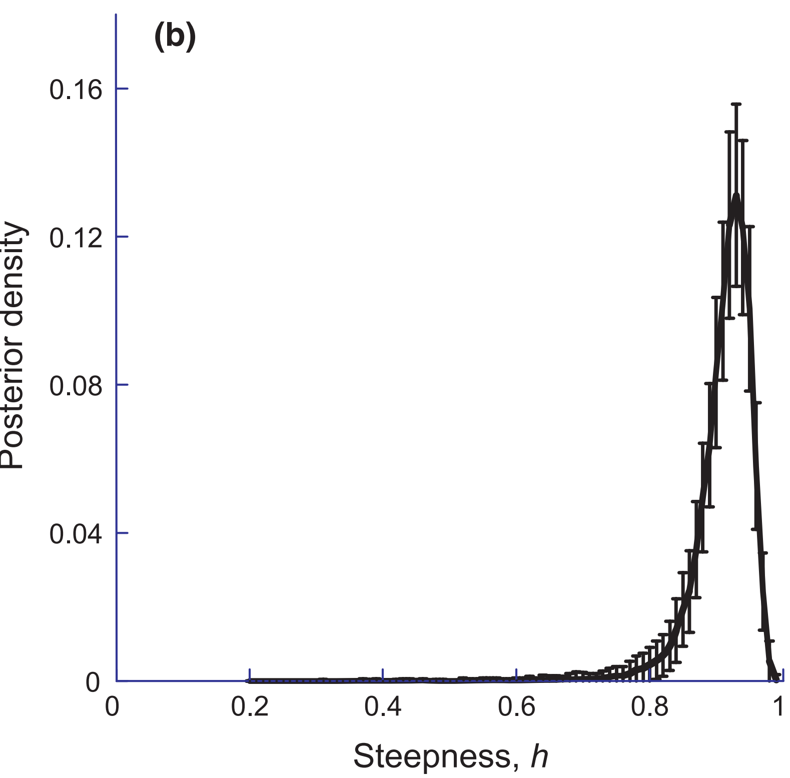

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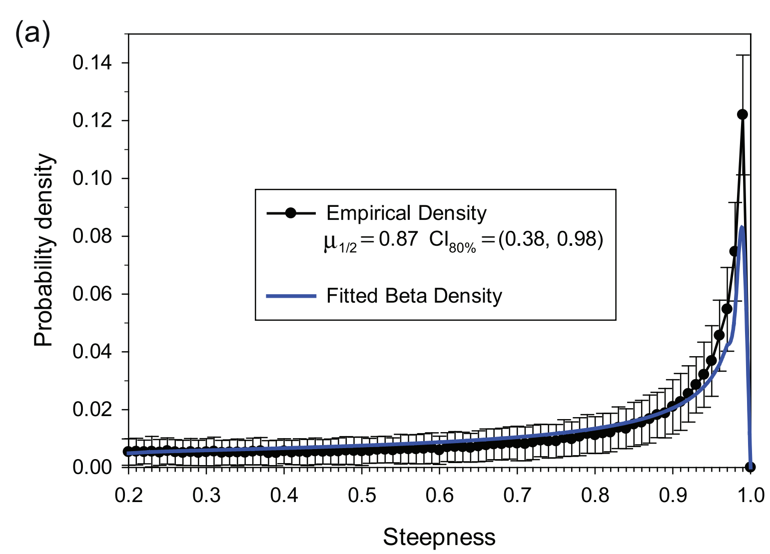

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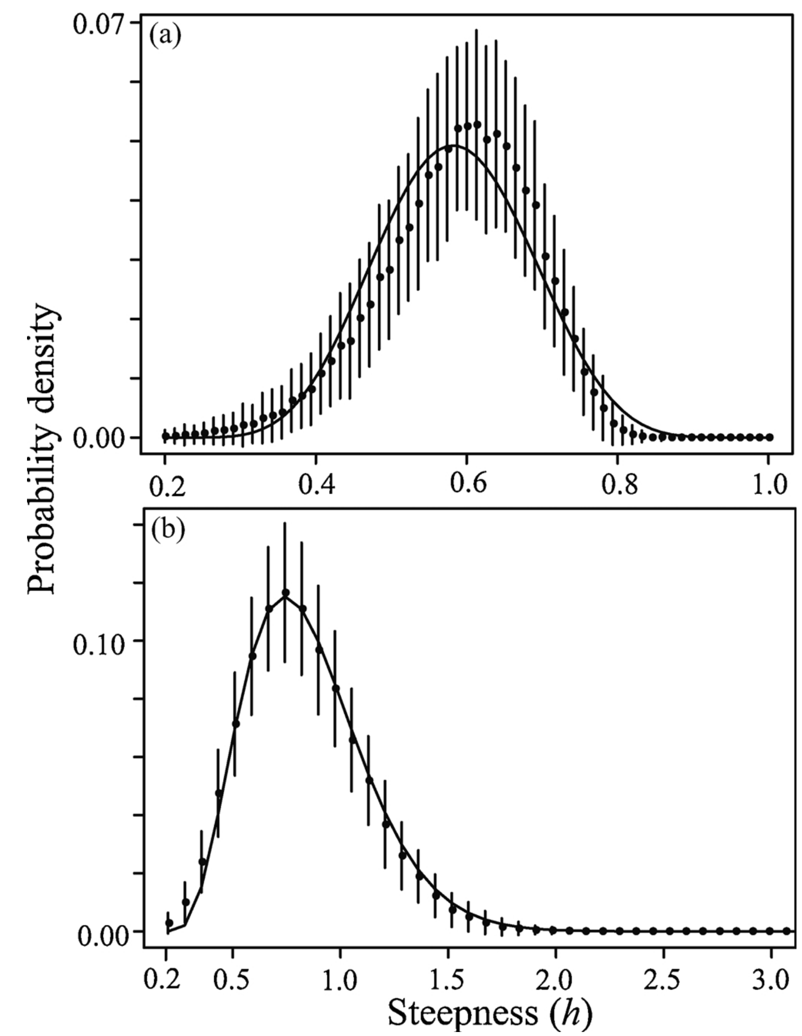

BH-SRR

Ricker-SRR

Blue shark
Kai and Fujinami Fish. Res. 2017

Thing #5: Specifying Both Natural Mortality and Steepness Will Lead to Problems, Or Worse

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Rate of mortality at age

$$h = \frac{\alpha \cdot \Sigma_{a=0}^{A_{max}} \prod_{a'=0}^{a-1} e^{-M(a')} L(a)^b p_m(a) \phi(a)}{4 + \alpha \cdot \Sigma_{a=0}^{A_{max}} \prod_{a'=0}^{a-1} e^{-M(a')} L(a)^b p_m(a) \phi(a)}$$

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Case of constant mortality: $M(a) = M$

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Case of Constant Mortality

$$h = \frac{\alpha \cdot \Sigma_{a=0}^{A_{max}} e^{-Ma} L(a)^b p_m(a) \phi(a)}{4 + \alpha \cdot \Sigma_{a=0}^{A_{max}} e^{-Ma} L(a)^b p_m(a) \phi(a)}$$

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<u>Case of Constant Mortality</u>

Okay to fix this

$$h = \frac{\alpha \cdot \Sigma_{a=0}^{A_{max}} e^{-Ma} L(a)^b p_m(a) \phi(a)}{4 + \alpha \cdot \Sigma_{a=0}^{A_{max}} e^{-Ma} L(a)^b p_m(a) \phi(a)}$$

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<u>Case of Constant Mortality</u>

Or this

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Case of Constant Mortality

But both?

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Best result

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Best result

Worst result

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Here's steepness (Mangel et al 2010, 2013)

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$$\frac{dN}{dt} = \frac{\alpha E_T}{1+\beta E_T} - MN$$ set $E_T = \phi N$ and $\alpha_p = \alpha\phi$, $\beta_p = \beta\phi$

$$\frac{dN}{dt} = \frac{\alpha_p N}{1+\beta_p N} - MN$$

$$N_0 = \frac{1}{\beta_p}\left(\frac{\alpha_p}{M} - 1\right)$$

Reason for writing the BH-SRR as above

Here's steepness (Mangel et al 2010, 2013)

$$h = \frac{\frac{\alpha_p}{M}}{4+\frac{\alpha_p}{M}}$$

Really sweet interpretation

Thing #7: Clear Reference Points Come From the Production Model

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$$SPR_{MSY} = \sqrt{\frac{1-h}{4h}}$$

Thing #7: Clear Reference Points Come From the Production Model

An Example

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An Example

PACIFIC COAST GROUNDFISH FISHERY MANAGEMENT PLAN

FOR THE CALIFORNIA, OREGON, AND WASHINGTON GROUNDFISH FISHERY

PACIFIC FISHERY MANAGEMENT COUNCIL
7700 NE AMBASSADOR PLACE, SUITE 101
PORTLAND, OR 97220
(503) 820-2280
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AUGUST 2016

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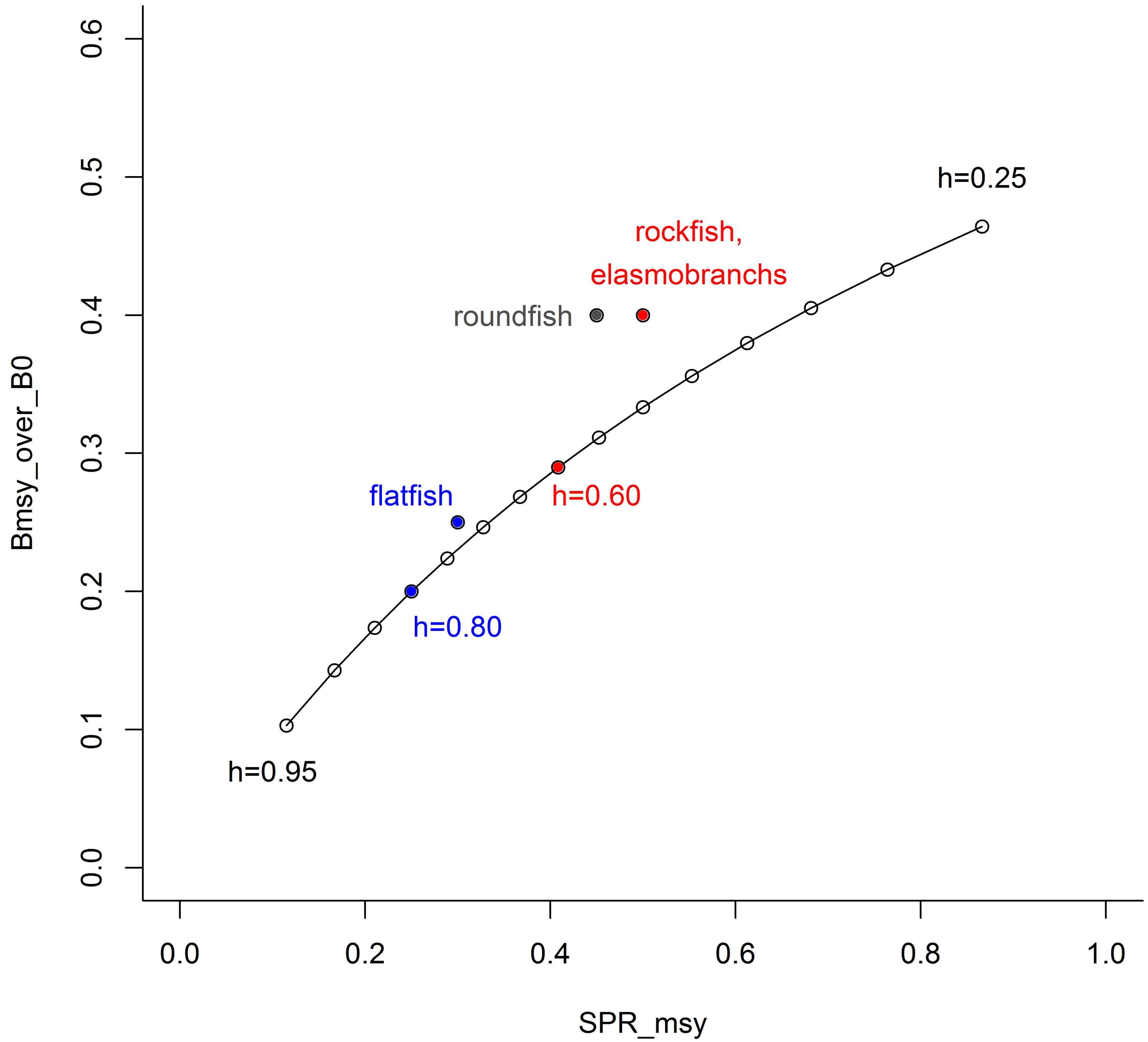

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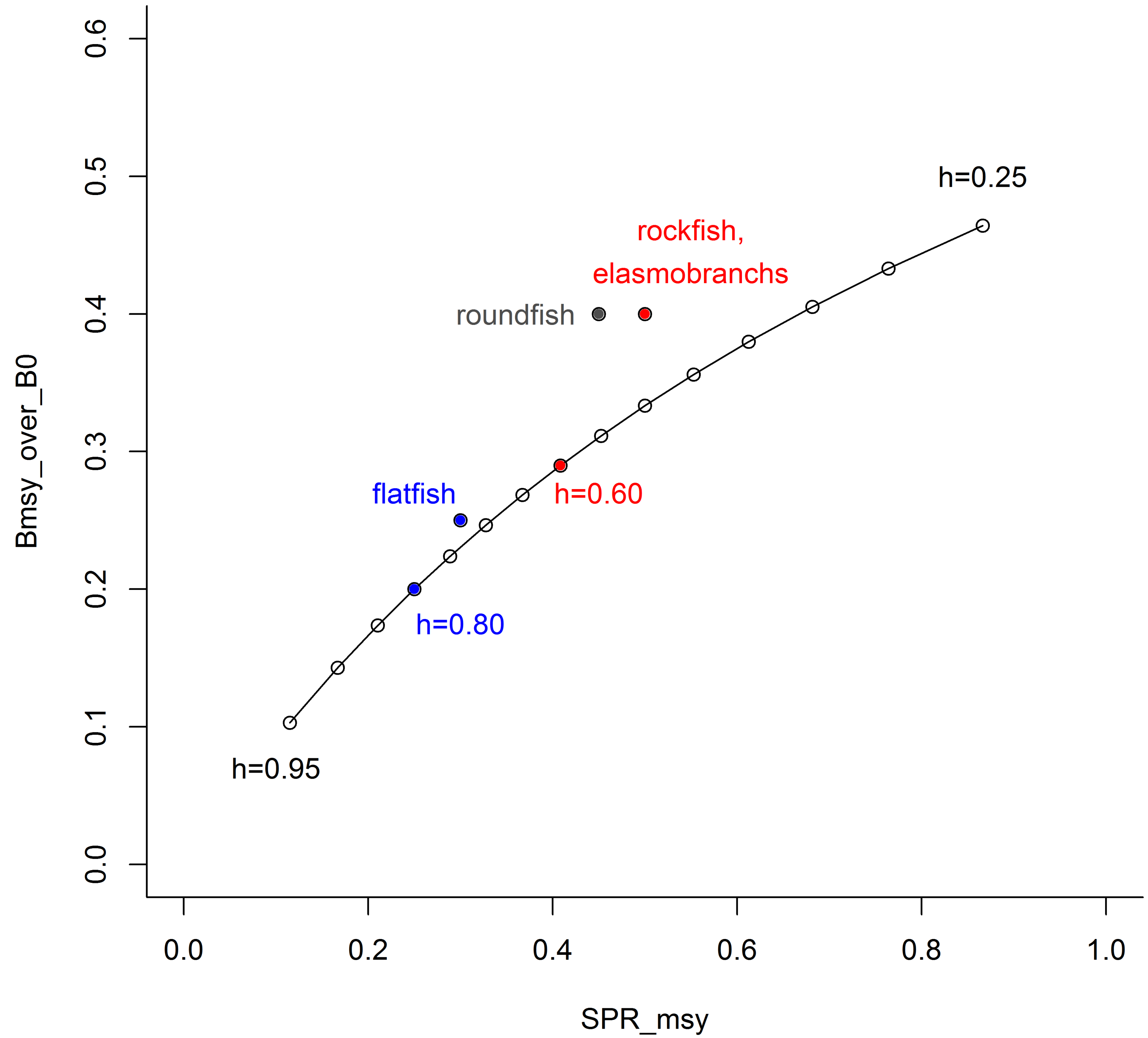

- Ongoing work (EJ Dick, Nick Grunloh, Mangel): W. How much of a difference will it make if a SRR is chosen that hits the reference points?

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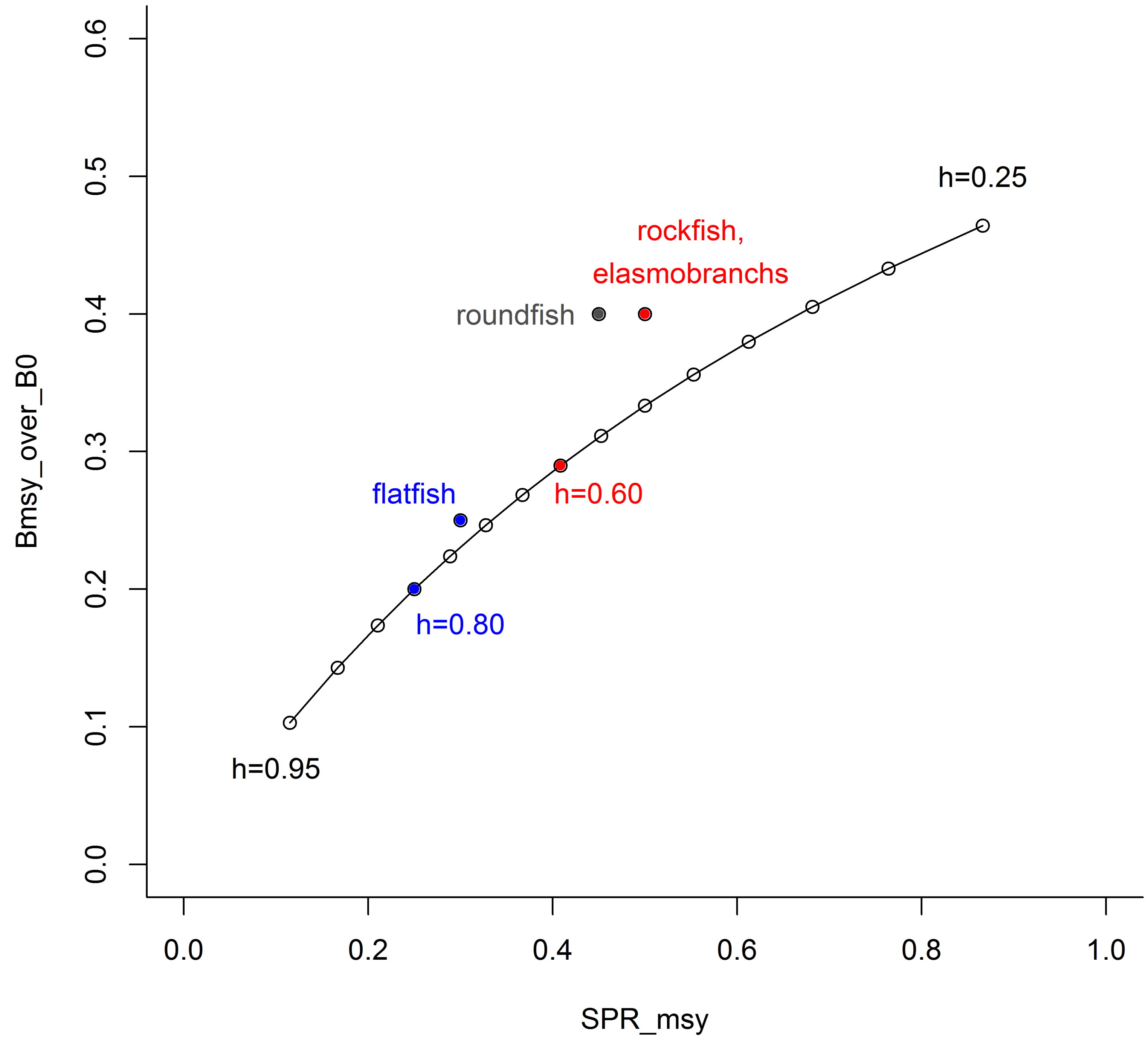

- Ongoing work (EJ Dick, Nick Grunloh, Mangel): W. How much of a difference will it make if a SRR is chosen that hits the reference points?
- How good an approximation is the production model?

Thing #8: Setting Steepness Equal to 1 Is About As Non-Conservative As You Can Get

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Robyn E. Forrest, Murdoch K. McAllister, Martin W. Dorn, Steven J.D. Martell, and Richard D. Stanley

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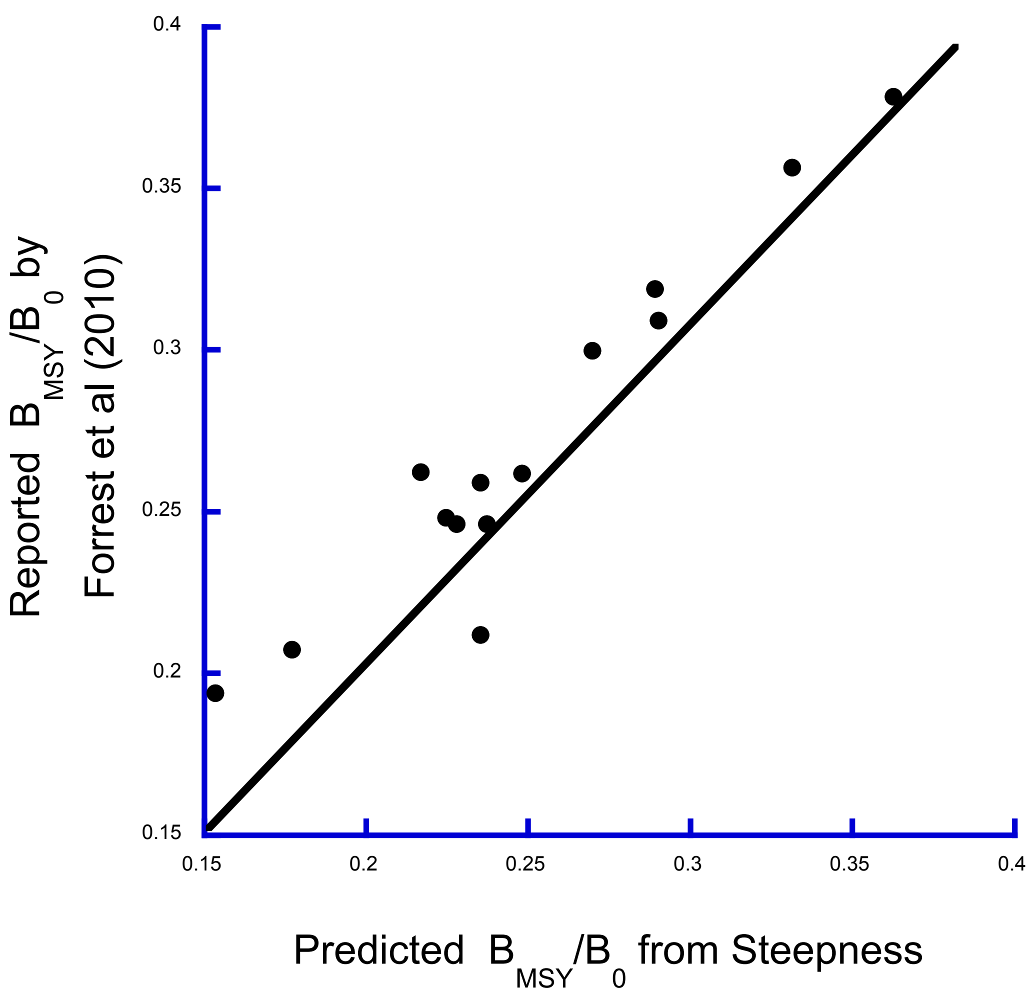

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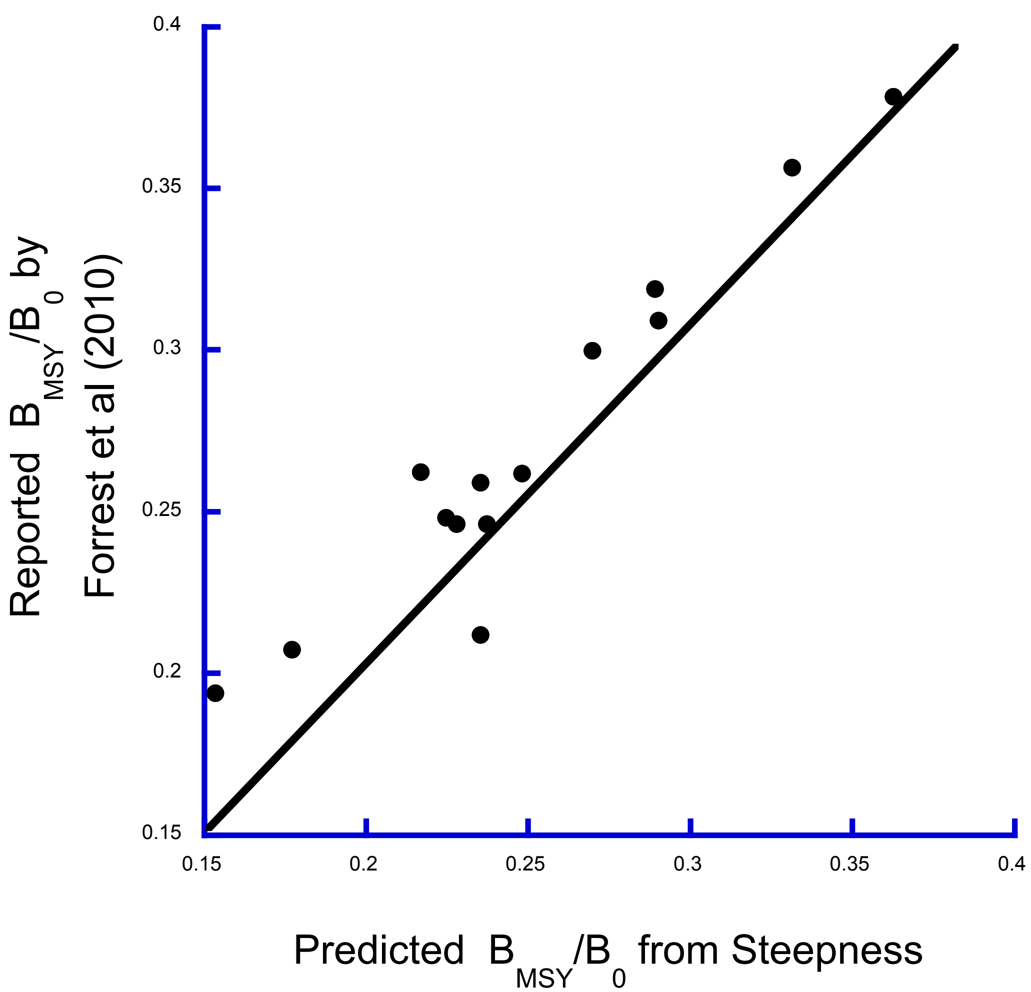

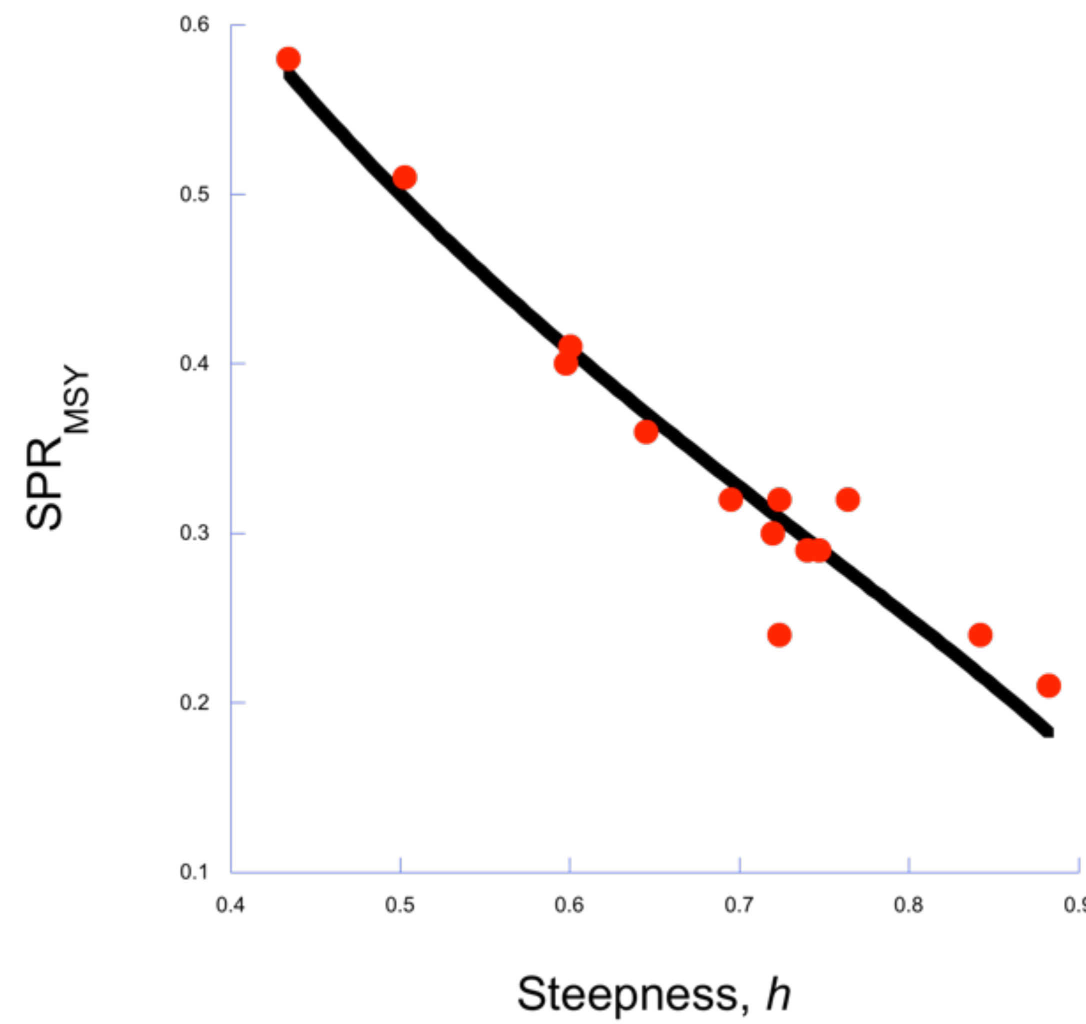

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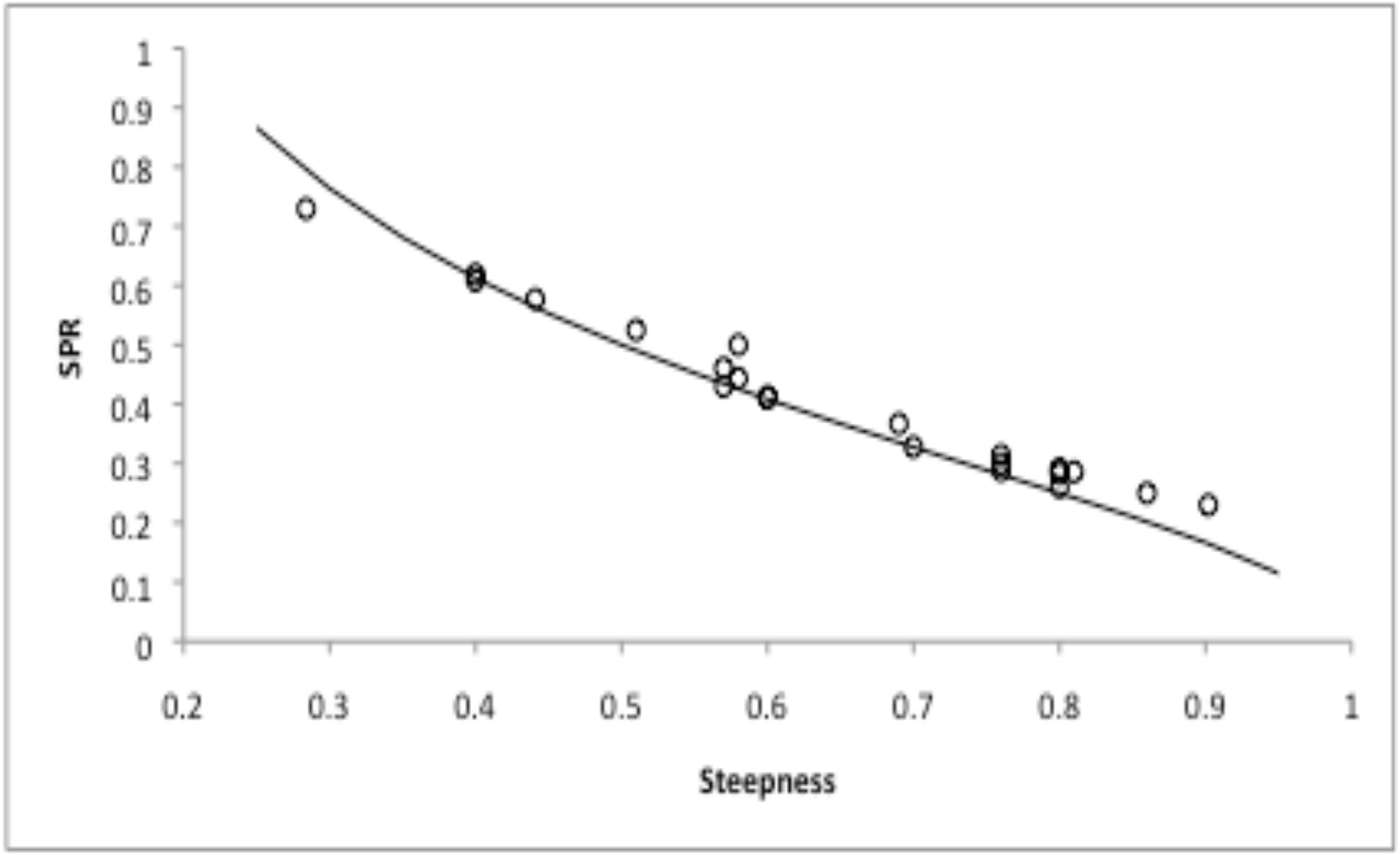

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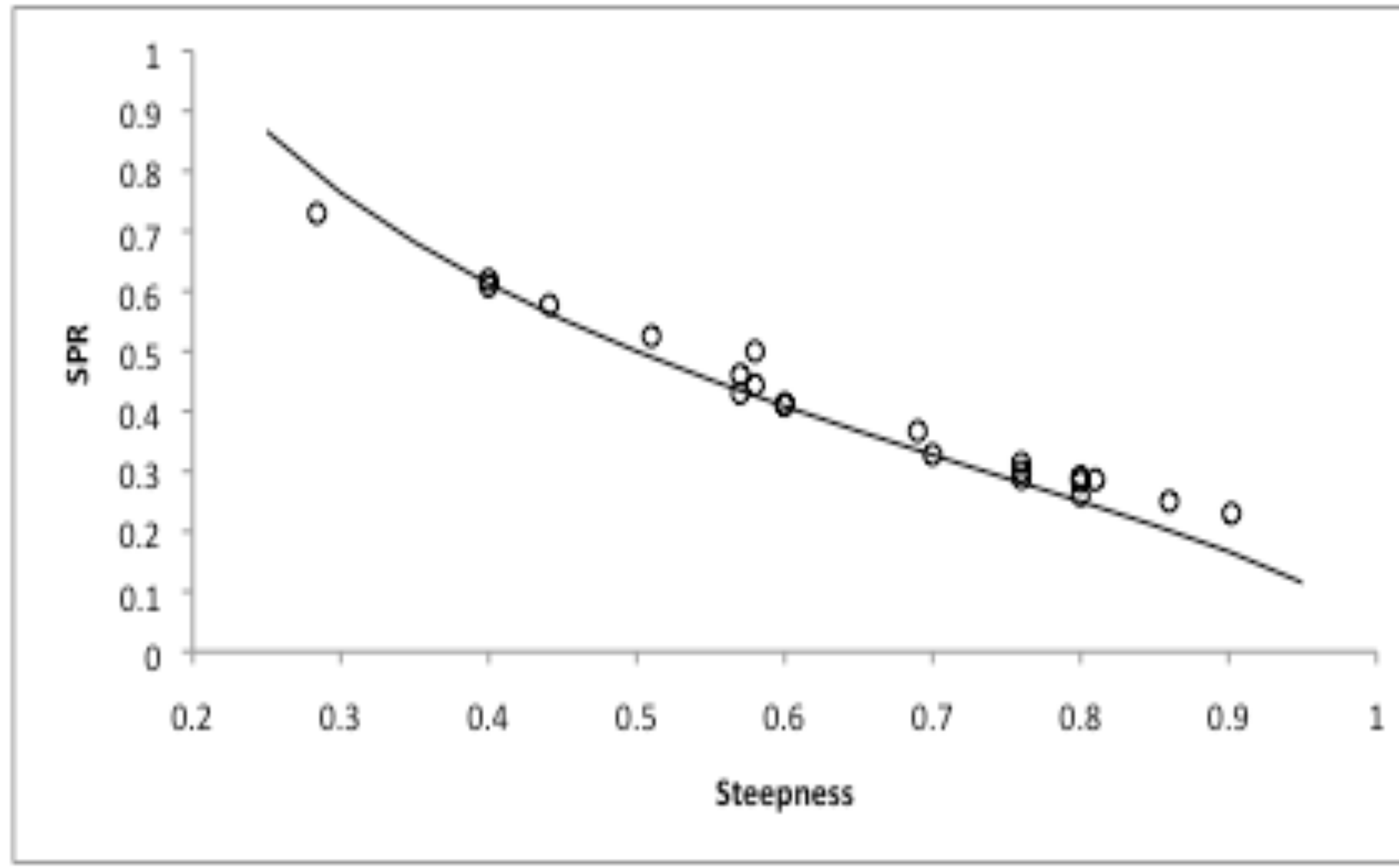

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Thing #10: Faster Growth Makes the Age-Structured Model More and More Like the Production Model

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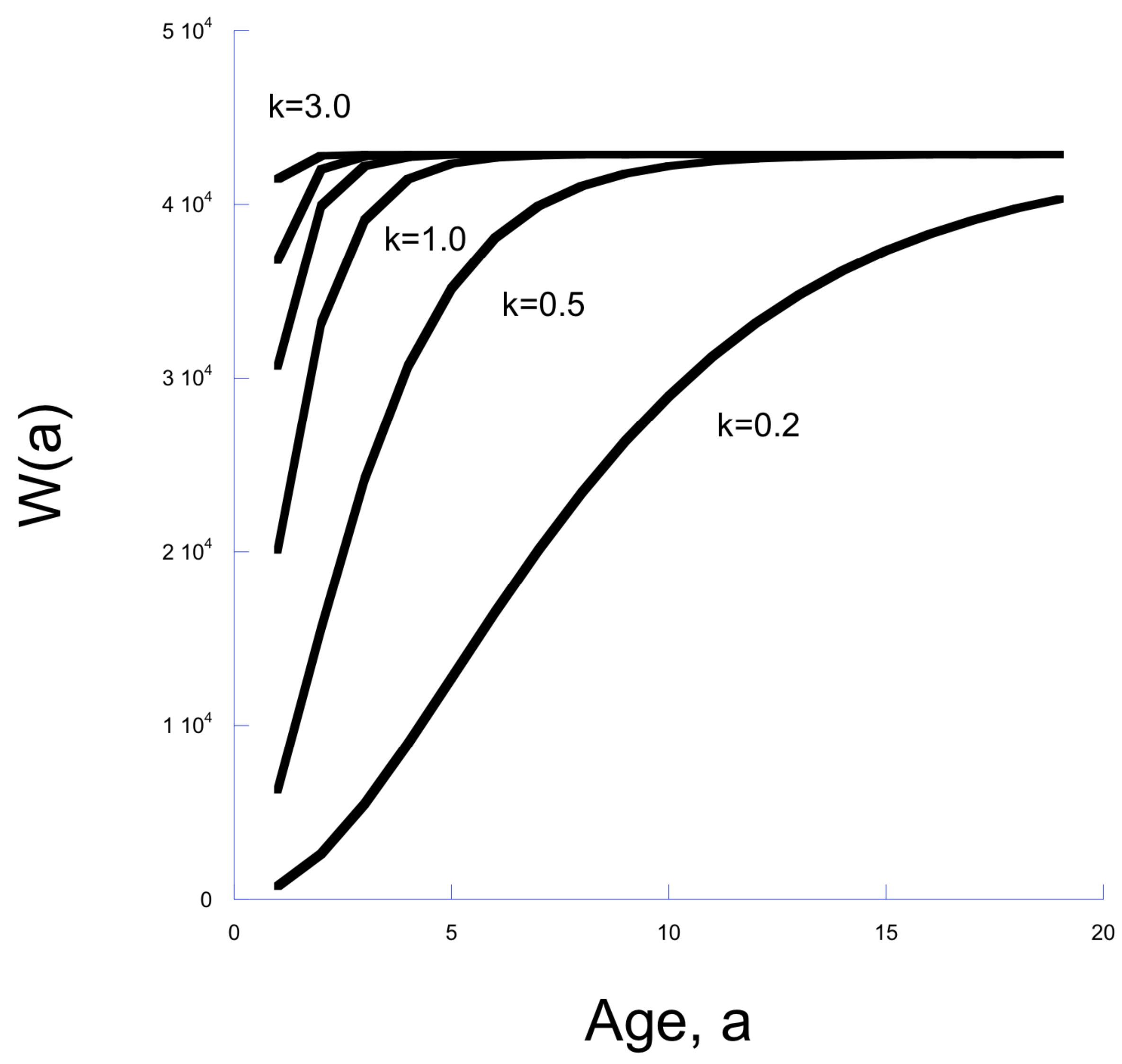

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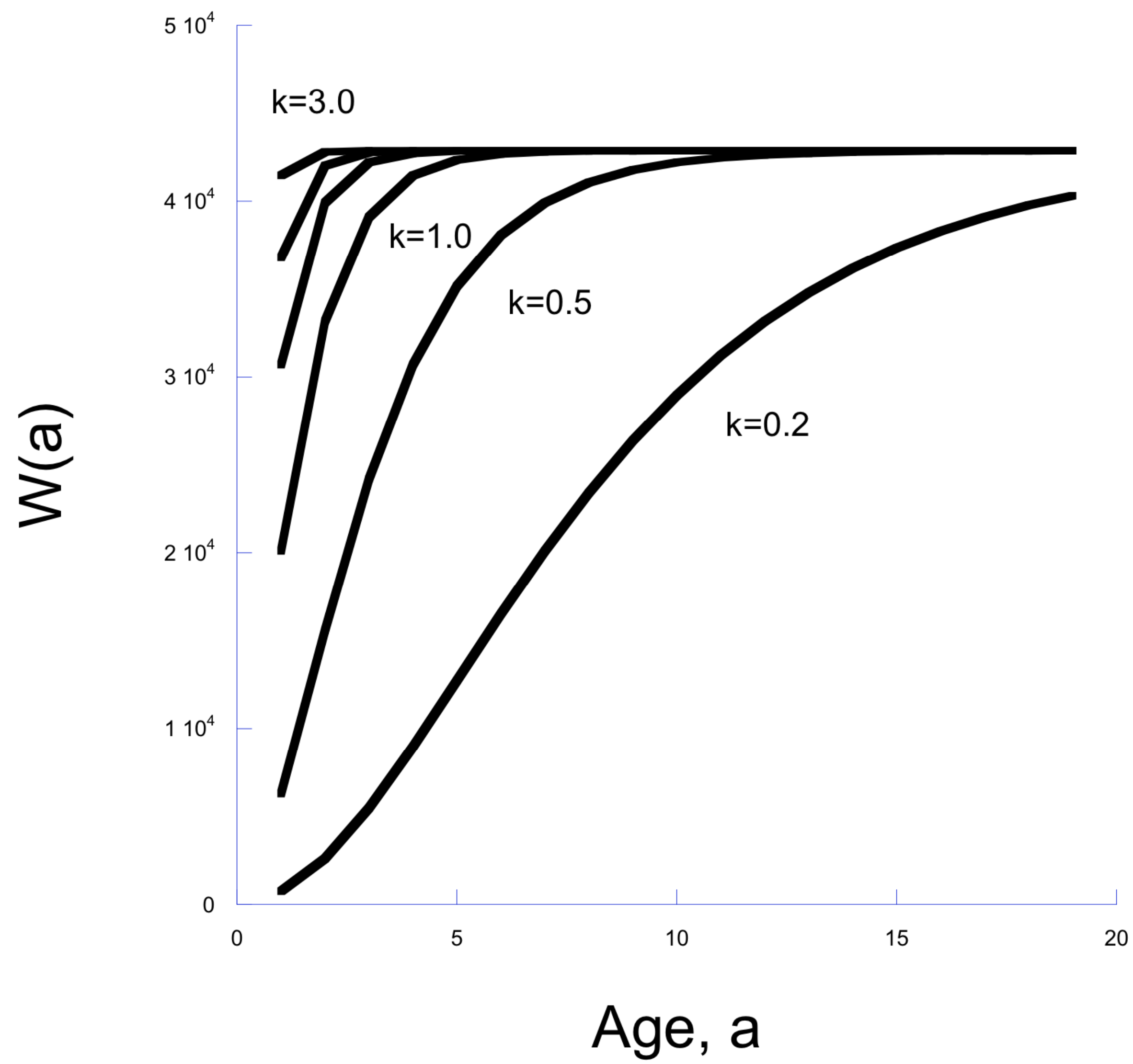

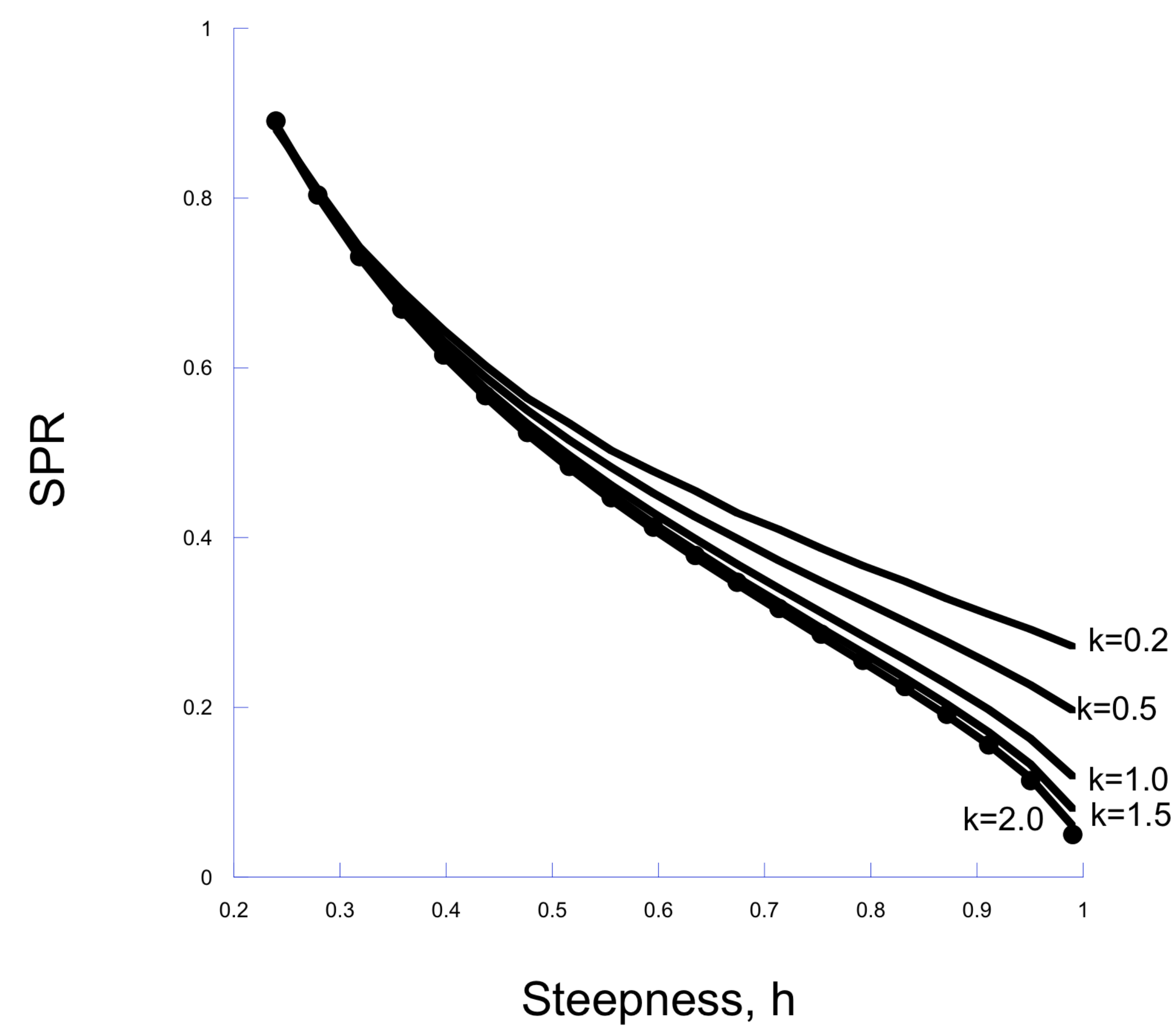

Thing #11, Bonus Track: If You Don't Know What Determines Recruitment, The Right Prior Is Uniform, Tied Down at 0.2 and 1

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<u>How to be Wrong</u>

$$h = 1$$

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$$h = 1 \quad \text{means} \quad \Pr\{R(0.2B_0) = R_0\} = 1$$

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With certainty, recruitment at 20% of unfished biomass is unfished recruitment. You know a lot about what determines recruitment.

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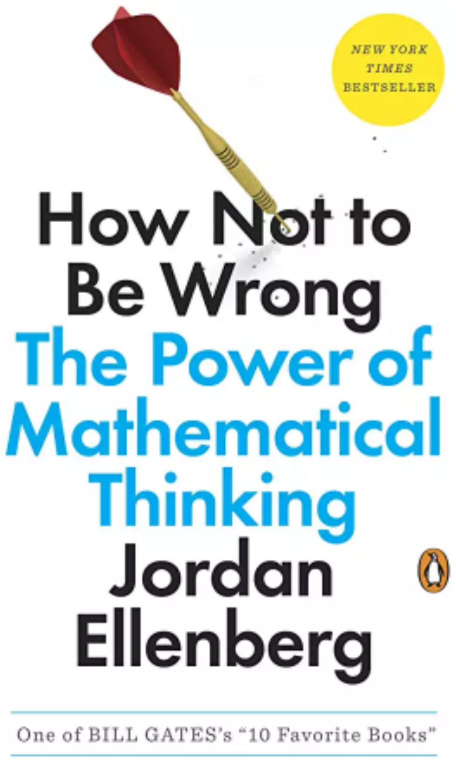

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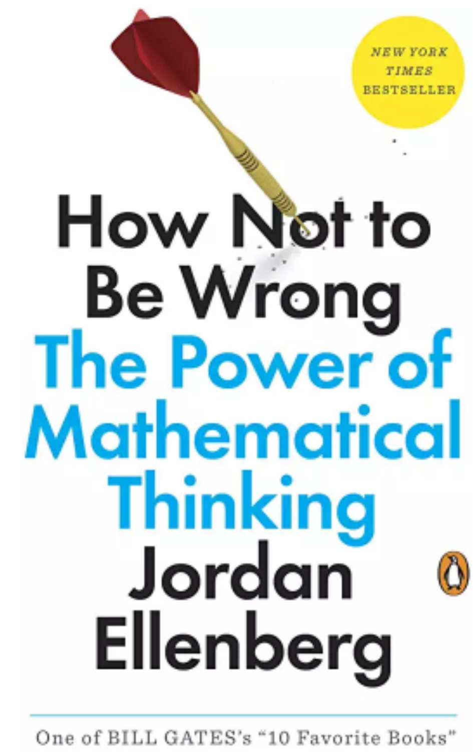

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If recruitment at 20% of unfished biomass can take any value between 20% of unfished recruitment and unfished recruitment, steepness ranges between 0.2 and 1.0 (tied down at both ends by biology).

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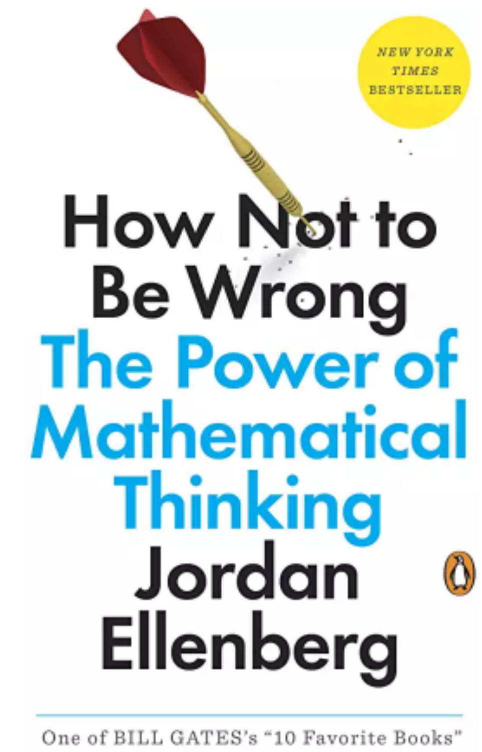

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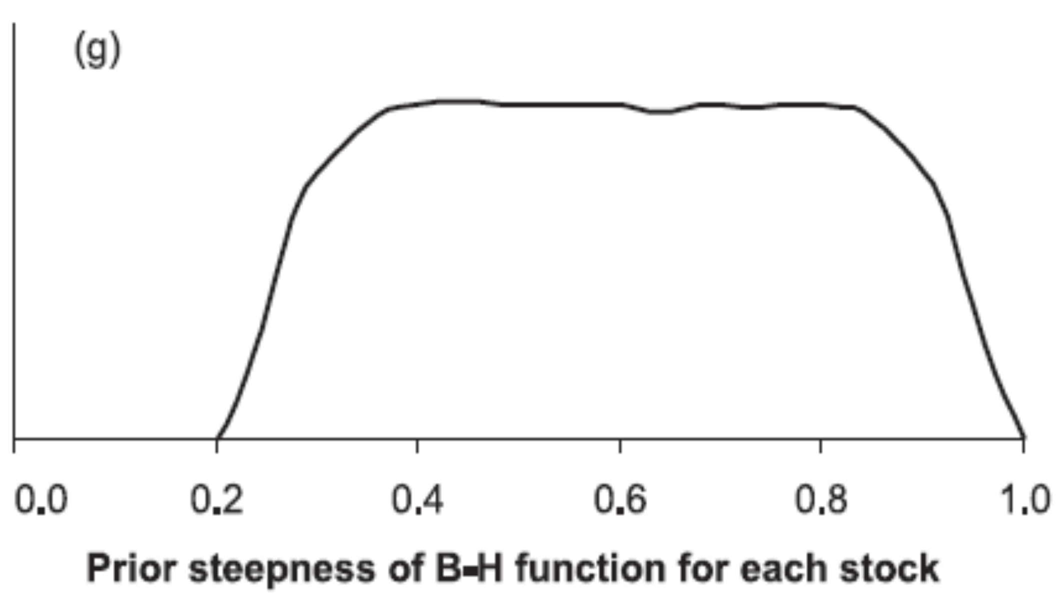

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Some Citations

2010 Mangel, M., Brodziak, J.K.T., and G. DiNardo. Reproductive ecology and scientific inference of steepness: a fundamental metric of population dynamics and strategic fisheries management. Fish and Fisheries 11:89-104. **Erratum**

2013 Mangel, M., MacCall, A.D., Brodziak, J., Dick, E.J., Forrest, R.E., Pourzand, R., and S. Ralston. A perspective on steepness, reference points, and stock assessment. Canadian Journal of Fisheries and Aquatic Sciences 70:930-940

2014 Brodziak, J., Mangel, M. and C-L Sun Stock-recruitment resilience of North Pacific striped marlin based on reproductive ecology. Fisheries Research, http://dx.doi.org/10.1016/j.fishres.2014.08.008

Options for Moving Forward

Do Not Fix Steepness and Mortality Rate

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Replace the BH-SRR by a SRR that Avoids the Problem

Do Not Fix Steepness and Mortality Rate

When will data be informative?

Use Simulation Methods to determine what kinds of data are necessary so that steepness and natural mortality can be estimated in the stock assessment

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Already started:

Lee, H.-H. et al. 2011. Estimating natural mortality within a fisheries stock assessment model: An evaluation using simulation analysis based on twelve stock assessments. Fisheries Research 109: 89-94.

Lee, H.-H. et al. 2012. Can steepness of the stock–recruitment relationship be estimated in fishery stock assessment models? Fisheries Research 125-126: 254-261.

Replace the BH-SRR by a SRR that Avoids the Problem

An example: Maynard Smith/Shepherd model

$$\frac{dB}{dt} = \frac{\alpha_p B}{1 + \beta B^{\frac{1}{n}}} - (M + F)B$$

$$\frac{F_{MSY}}{M} = \frac{\frac{\alpha}{M}(1-n) + \sqrt{\left(\frac{\alpha}{M}\right)^2 (1-n)^2 + 4\frac{\alpha n}{M}}}{2} - 1$$

There is still an undetermined parameter for the analysis -- the data can tell us something! -and if not
we need to integrate over the potential range of *n*